Reflection as a Learning Tool in Mathematics

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Undergraduate students in developmental math classes tend to have a narrow view of what mathematics is and fail to see its connection to the world around them. For many such students, math is only a set of formulas that they must learn and manipulate to obtain numerical answers. Contributing to this situation is the way in which math is traditionally taught: Techniques and procedures are emphasized, and little attention is paid to the purposes behind the formulas or to the interpretation of numerical results. In discussing a comparative study of math classes in seven countries and the factors that correlate with student success, Watson and De Geest discount the importance of superficial aspects [of pedagogy] such as whether teaching is whole-class or not, students work in groups or not, students discuss together or not, how the board is used, when homework is discussed and so on. The common features of successful countries appear to be more subtle, the most significant characteristic being the way that mathematical concepts are presented. The complexity of the concepts and methods is preserved, rather than simplified, in the ways teachers work. (213)

In the workplace and in everyday life, many problems are best addressed through logic and quantitative analysis, but they are often too complex to be reduced to the form of a typical math problem. As a result, I have come to believe that reflection is the most valuable tool for developing the ability of students to use mathematical concepts to address a variety of practical concerns and to engage in deeper thought about the issues involved.

The Limitations of Procedural Knowledge
Many math classes focus primarily, or even exclusively, on procedural knowledge, which means the ability to carry out step-by-step calculations using certain formulas and mathematical operations (addition, multiplication, etc.) to arrive at a numerical or algebraic answer to the
problem at hand. To solve a math problem in this way, students first have to translate the statement of the problem into mathematical concepts, then use math procedures to arrive at a numerical or algebraic solution, and finally interpret the solution in a way that answers the question posed in the problem. This traditional approach to mathematics can be illustrated with the following problem, which is typical of those encountered in a basic math course and which can be solved with simple procedural knowledge:

Problem: Sugar is to be packaged in 200-, 250-, or 400-gram packets. Determine the smallest quantity of sugar required so that any of the three sizes of packet can be used, all packets will be completely filled, and no sugar will remain unpacked. Also indicate the number of packets of each size that is needed to handle this quantity of sugar.

To solve this problem, students must first translate the statement of the problem into a mathematical procedure. In this case, an exact number of packets is needed, so students must look for multiples of 200, 250, and 400. So that packets of any of the three sizes can be used, the students must then look for common multiples; and to ensure that the smallest quantity of sugar is used, they need to determine the Least Common Multiple (LCM). This step-by-step procedure leads to an LCM of 2000. Finally, to arrive at the number of packets of each size that is needed, the students must divide the LCM by 200, 250, and 400, with the answer to the last part of the problem being 10, 8, and 5, respectively.

Students tend to find this type of problem boring and unimaginative, and it leads them to view the study of math as little more than the mastery of procedural steps. What students fail to see, and what math teachers often fail to teach, is the application of procedural knowledge to real-life situations. In the preface to Calculus, Hughes-Hallett et al. state:

Calculus has been so successful because of its extraordinary power to reduce complicated problems to simple rules and procedures. Therein lies the danger in teaching calculus: it is possible to teach the subject as nothing but rules and procedures – thereby losing sight of both the mathematics and its practical value. (v)
Math instructors cannot ignore procedural knowledge since, without it, students cannot build the confidence to handle real-life problems; procedural knowledge alone, however, is useless if students do not develop the skill to apply it. Both skill areas must be integrated in mathematics lessons.

A second example of the limitations of procedural knowledge can be found in a typical lesson from an elementary statistics class. When students are introduced to the concept of standard deviation (the average deviation from the center of the data), the teacher will usually give a definition (a measure of the spread of data values), provide the formula to compute standard deviation, and use a set of numbers to show the computations step by step. Problems from a textbook may then provide some practice. It is rare, however, for students to see the application of such procedural knowledge. If, for example, they are given commute-time data for two trains and asked to select the more dependable train, most students will calculate the average commute time for each train and select the one with the lower average. If the average commute times for the two trains are the same, most students will not think to take the problem a step further and use standard deviation as a helpful tool in making their decision.

While the need to apply procedural knowledge is very common in the workplace, most tasks do not come in the form of a traditional math problem. In most cases, the statement of a problem is in the common language of the trade, but the solution required must be expressed with the precision of math. In the workplace, there is a great demand for employees who are able to understand and reflect on practical issues using math. The question thus arises: How can math instructors prepare their students for the demands and challenges of today’s workplace?

The Power of Reflection
As early as 1910, Dewey discussed the importance of using reflection as a learning tool. In *How We Think*, he defined reflective thinking as the examination of one’s beliefs, the evidence that supports them, and the conclusions to which they lead (7). He argued that a crucial component of reflective thinking is the adoption of an attitude of suspended judgment and the mastery of various methods of seeking out evidence to support or refute the first suggestions that come to mind (12). In *Democracy and Education*, Dewey further explained that the objective of reflective thinking is greater “efficiency of action” as well as “learning more about ourselves and the world in which we live” (152).
Silcock expands on Dewey’s ideas in defining reflection as “a ubiquitous, cognitive process, not only reworking tacit knowledge into skill, but providing, through symbolic transformations, a means for linking social and knowledge-contexts, and for translating one sort of experience (e.g. academic) into another (e.g. practical)” (274). Thus, in a mathematical context, reflection involves examining the procedural knowledge used in everyday practice in such a way that its application can be broadened beyond immediate circumstances.

Both Wheatley and Sigel describe reflection in math as a process of “distancing oneself from the action of doing mathematics (Wheatley (citing Sigel) 535).” As Wheatley states:

In the process of reflection, schemes of schemes are constructed – a second-order construction. Persons who reflect have greater control over their thinking and can decide which of the several paths to take, rather than simply being in the action.

It is not enough for students to complete tasks; we must encourage students to reflect on their activity. (535)

Banker agrees that reflective pedagogy in the mathematics classroom leads to a deeper understanding of the subject matter:

Those teachers who provide opportunities for students to communicate about mathematics cultivate a supportive, nonthreatening environment that can deepen the understandings that students need to make mathematics meaningful and to connect informal ideas about mathematics to the symbols and notation in the language of mathematics. (2)

Evans has discussed the strengths and weaknesses of various teaching approaches geared towards connecting mathematical abstractions with everyday life.

Despite the advantages of a reflective approach, relatively few math educators make reflection a central part of their classroom pedagogy. My own goal in incorporating reflection-based activities into the curriculum is to help my students to draw on their own ability to reason, to examine the applications of procedural knowledge, and, thereby, to develop into mathematical thinkers.
Toward a Reflective Math Pedagogy
With these ideas in mind, I have designed and used reflection-based activities in my mathematics classes. Encouraged to think beyond formulas and numbers, students will learn to use mathematics as a tool to resolve real-life problems and to articulate the thinking processes that they apply to solving such problems.

Environmental issues have provided a rich source of material for reflection by my math students. In one activity, the students learned from their readings that the percentage of greenhouse gases in the earth’s atmosphere (including carbon dioxide) is less than 1% (Hanson). They then examined a pie-chart on the official website of the United States Energy Information Administration (“U.S.”) which showed all greenhouse gases and indicated that the percentage of carbon dioxide is 82%. Classroom discussion focused on the following question: Does the information on the pie-chart mean that the earth’s atmosphere is filled with carbon dioxide and that it contains very little oxygen or other gases? This question required students to think carefully and critically in order to understand the discrepancy between two seemingly contradictory numbers: The percentage of greenhouse gases is 1%, while the percentage of carbon dioxide, which is a greenhouse gas, is 82%. In order to resolve the apparent contradiction, the students had to synthesize the information given about greenhouse gases and reflect on their understanding of the mathematical concept, a percent of a percent. In this case, the percentage of carbon dioxide in the earth’s atmosphere is 82% of 1%, which is only 0.82% (less than 1%) of the total amount of all gases in the atmosphere. A second question was then posed: Why should scientists worry about greenhouse gases if the total amount of these gases is less than 1%? In the ensuing discussion, the students came to realize that there is a delicate balance among the gases in the earth’s atmosphere and that even a slight change in the amount of a gas such as carbon dioxide can have a significant impact on the earth’s climate.

In another assignment, used in a basic math class as part of Project Quantum Leap (PQL)1, students learned from an article on “Global Warming,” that “the average surface temperature of the earth [has risen] by more than 1 degree Fahrenheit since 1900.” They then looked at a second article which mentioned that the widespread use of asphalt and concrete has turned New York City into an “urban heat island,” in which nighttime temperatures are estimated to have risen by 7 degrees Fahrenheit over the past century (Tierney). The students were asked...
to compare what they had read in the two articles and to discuss the apparent contradiction between the numerical facts. They were also asked to use numerical information to support their argument.

This example presents various opportunities for reflective thinking since it involves a number of commonly-held beliefs. First of all, most students believe that the phenomenon of global warming implies higher temperatures everywhere, but this is not, in fact, the case. Secondly, for most students, average means the result of adding all the values and then dividing the sum by the number of values. If the average temperature rise is 1 degree, which is a small number, most students would not be aware that an individual temperature rise could be as high as positive 7 degrees. It is also less than obvious that if the average temperature is a positive value, the individual values could be a mix of positive and negative values. In this assignment, therefore, students discover that average is the equal distribution of the sum total. It is possible to have a 7-degree rise in temperature in one place and a 5-degree drop in temperature in another place, and the average will still be a rise in temperature of 1 degree. This is also an opportunity for students to discuss whether they find average to be a meaningful measure in a situation in which action needs to be taken to control the rise in temperature.

When I use reflective assignments in class, my own role as teacher becomes one of posing a series of guiding questions in order to lead the students toward a greater understanding of both the mathematical concepts and the real-life issues that are involved. In the above example, I guide the students with questions such as these: What are the numerical facts that are presented in the articles? What is the process for finding the average surface temperature of the earth? What does it mean if the average surface temperature of the earth increases by 1 degree Fahrenheit over the course of a century? How can the average rise in temperature be 1 degree when the temperature has risen by 7 degrees in a single location? Does global warming mean that temperatures rise everywhere? In a period of global warming, how can temperatures in some places decrease rather than increase? These are the types of questions that help students to deepen their understanding of mathematical concepts and to use these concepts to understand a real-life issue. At the same time, they are engaging in a lively discussion that helps to motivate them and to keep the level of enthusiasm high.

I have noticed, however, that for some students, the increased level of motivation continues while, for others, the motivation is only for a limited time. Why? I think the answer to this question is that reflection-
based activities cannot be used in isolation but need to be part of every lesson and integrated into the entire curriculum in order to have a major impact on student learning and success. The short-term benefits of reflective pedagogy may be limited, but the long-term benefits of a sustained approach can be significant.

Developing a curriculum that focuses on reflection-based pedagogy involves its own set of challenges, not the least of which involves the question of assessment. The standardized tests in current use neither evaluate nor encourage reflective thinking on the part of students. Teaching in such a system places great pressure on instructors since they must not only complete the predetermined syllabus but also prepare their students to succeed on the standardized tests, all within strict time limits. If we want to foster reflection in the classroom, we need to develop new tools for assessment that are more relevant for evaluating the skills that are required in the students’ subsequent professional careers. As articulated by Dapueto and Parenti,

it is necessary to outline a new set of basic skills and a new balance between experiential and reflective learning, to develop new criteria for the assessment which exploit the opportunity for dynamic evaluation (day-to-day, in various activities,...) that differs from pre-set and ad hoc testing, but overcome the risk of prejudicial evaluations. (19, emphasis in orig.)

An example of such “dynamic evaluation” would be open-ended project assignments that encourage students to demonstrate the depth of their understanding of mathematics rather than simply arrive at a single correct answer. Not only would this form of evaluation be more meaningful, but it would also more closely resemble the types of problems actually encountered in the workplace. Similarly, student ePortfolios can also be used for evaluation purposes by both educational institutions and prospective employers (“About ePortfolio”). Having students explore an exam topic in the form of an essay could also be a useful form of evaluation since writing requires students to express a deeper understanding of the topic, encourages them to engage in higher-order thinking, keeps teacher expectations high, and makes learning more challenging.

For most students, participating in a reflection-based math class brings challenges faced most probably for the very first time, and a sin-
gle semester does not necessarily provide enough time to demonstrate improvement. Nevertheless, being trained to reflect on the applications of math to real-life problems should help students in their future studies and careers. Such an approach deepens knowledge by asking students to examine the facts, to draw on fundamental concepts of math to respond to specific questions, to use appropriate formulas to arrive at logical results, and finally, to modify their understanding not only of the relevant mathematical concepts but also—and equally important—of the real-world problems that math can help to address and resolve.

Notes
1. PQL is a FIPSE (Fund for the Improvement of Postsecondary Education)-funded program run by the LaGuardia Center for Teaching and Learning to develop curriculum materials for teaching basic-skills math courses using the SENCER (Science Education for New Civic Engagements and Responsibilities) approach. SENCER is a National Science Foundation (NSF)-funded project that promotes the development of courses that teach science and advanced mathematics through “compelling contexts,” i.e., complex, capacious, and unsolved public issues.

Works Consulted


