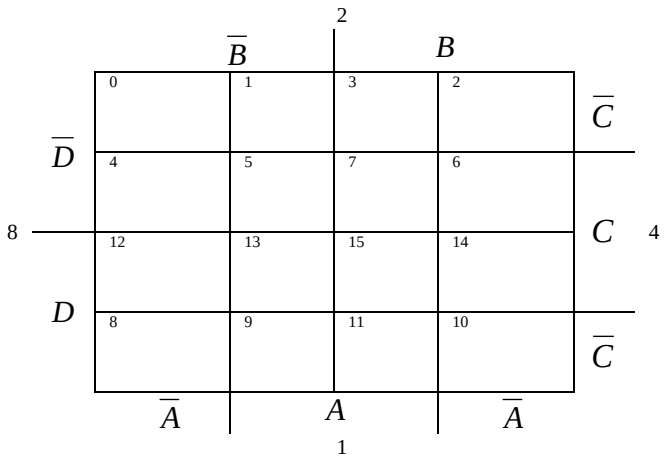
The Brumgnach-Karnaugh Method for simplifying combinations of minterms.

1) Set up a Karnaugh Map with the following structure:



2) Write down each group of minterms (g1, g2, etc) using the minterm numbers

3) For each group of minterms write down the FIRST minterm (m0, m1, etc) using variable names.

   Actually, any of the minterms of the group could be used. The first minterm in each group was chosen for the sake of uniformity.

4) For each group of minterms:
   a. check for a difference of 8 between any two listed minterms. If there is one, cross out the variable with weight 8.
   b. check for a difference of 4 between any two listed minterms. If there is one, cross out the variable with weight 4.
   c. check for a difference of 2 between any two listed minterms. If there is one, cross out the variable with weight 2.
   d. check for a difference of 1 between any two listed minterms. If there is one, cross out the variable with weight 1.

5) Write the simplified equation by “ORing” the results obtained in step 4.

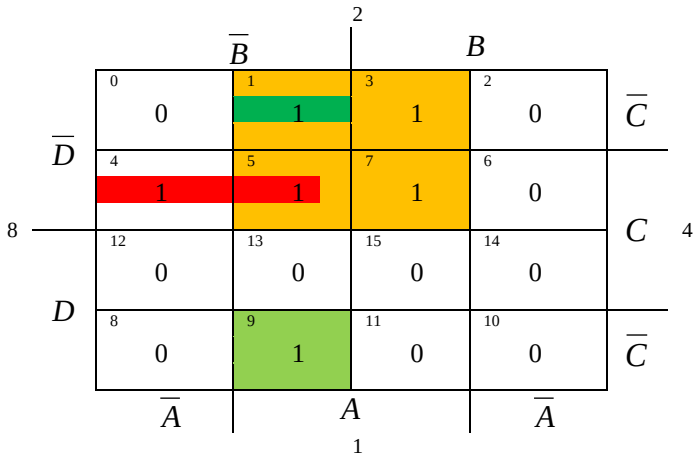
Example: In the truth table for a hex to seven segment decoder using a common anode seven segment display, the e segment of the display is high 6 times; at minterms 1,3,4,5,7, and 9.

Step 1: The equations in minterm form and variable form are

e = m1 + m3 + m4 + m5 + m7 + m9

$$e = \overline{D}\,\overline{C}\,\overline{B}\,A + \overline{D}\,\overline{C}\,B\,A + \overline{D}\,C\,\overline{B}\,\overline{A} + \overline{D}\,C\,\overline{B}\,A + \overline{D}\,C\,B\,A + D\,\overline{C}\,\overline{B}\,A$$

The Karnaugh map is



Step 2: Following the rules of forming groups on a Karnaugh Map, the following groups can be made $g1 = \sum(\text{m1}, \text{m3}, \text{m5}, \text{m7})$; $g2 = \sum(m4, m5)$; $g3 = \sum(m1, m9)$ where g1 stands for group 1, etc.

Step 3: For each group the FIRST minterm is as follows.

For g1, m1= $\overline{D}\,\overline{C}\,\overline{B}\,A$; for g2, m4=$\overline{D}\,C\,\overline{B}\,\overline{A}$; for g3, m1= $\overline{D}\,\overline{C}\,\overline{B}\,A$.

Step 4: For each group check for a numerical difference of 8, 4, 2, or 1 between any two listed minterms and cross out the variable with that weight.

For g1, 3-1=2, cross out the weight 2 variable (B); 5-1=4, cross out the weight 4 variable (C); leaving g1 = $\overline{D}\,A$

For g2, 5-4=1, cross out the weight 1 variable (A); leaving g2 = $\overline{D}\,C\,\overline{B}$

For g3, 9-1=8, cross out the weight 8 variable (D); leaving g3 = $\overline{C}\,\overline{B}\,A$

Step 5: The simplified equation is $e = \overline{D}\,A + \overline{D}\,C\,\overline{B} + \overline{C}\,\overline{B}\,A$

Simplification using Boolean Algebra

e = m1 + m3 + m4 + m5 + m7 + m9

$e = \bar{D}\,\bar{C}\,\bar{B}\,A + \bar{D}\,\bar{C}\,B\,A + \bar{D}\,C\,\bar{B}\,\bar{A} + \bar{D}\,C\,\bar{B}\,A + \bar{D}\,C\,B\,A + D\,\bar{C}\,\bar{B}\,A$

In each of the following combinations use:

FACTORING and the COMPLEMENT LAW ($\bar{x} + x = 1$)

1) combine m1 and m3 => $\bar{D}\,\bar{C}\,\bar{B}\,A + \bar{D}\,\bar{C}\,B\,A = (\bar{B} + B)(\bar{D}\,\bar{C}\,A) = \bar{D}\,\bar{C}\,A$
2) combine m5 and m7 => $\bar{D}\,C\,\bar{B}\,A + \bar{D}\,C\,B\,A = (\bar{B} + B)(\bar{D}\,C\,A) = \bar{D}\,C\,A$
3) combine m1, m3 and m5, m7 => $\bar{D}\,\bar{C}\,A + \bar{D}\,C\,A = (\bar{C} + C)\,\bar{D}\,A = \bar{D}\,A$
4) combine m4 and m5 => $\bar{D}\,C\,\bar{B}\,\bar{A} + \bar{D}\,C\,\bar{B}\,A = (\bar{A} + A)\,\bar{D}\,C\,\bar{B} = \bar{D}\,C\,\bar{B}$
5) combine m1 and m9 => $\bar{D}\,\bar{C}\,\bar{B}\,A + D\,\bar{C}\,\bar{B}\,A = (\bar{D} + D)\,\bar{C}\,\bar{B}\,A = \bar{C}\,\bar{B}\,A$
6) $e = [(m1, m3, m5, m7) + (m4, m5) + (m1, m9)]$
7) $e = \bar{D}\,A + \bar{D}\,C\,\bar{B} + \bar{C}\,\bar{B}\,A$