

1 Translations 1.6

By the end of this section, you should be able to solve the following problems.

1. Change the following to an algebraic expression, and use x to represent the unknown.

Five added to the product of a number and two.

2. Identify the following as an equation or an expression.

$$3x^3 - 57x + y$$

3. Fill in the blank with one of the following symbols to make the statement true: $=$, \neq , $<$, \leq , $>$, \geq .

$$5^2 \text{ ____ } 49$$

4. Determine if the following statement is true or false.

$$1 + \frac{2}{3} + 4\frac{1}{3} = 6$$

5. Evaluate the expression for the given value of the variable.

$$3m - 4m^2 + 1 ; m = \frac{1}{2}$$

2 Concepts and Examples

The words that indicate addition in the English language are:

plus, add, increase, and more.

Whenever we see these words in sentence we can translate them into the language of algebra by replacing them with a plus sign. Words that indicate subtraction are:

decrease, subtract, minus, take away, difference, and reduce

Whenever we see these words in sentence we can translate them into the language of algebra by replacing them with a minus sign. Words for multiplication are:

times, product, and multiply,

and examples of words that indicate division are:

quotient, and divide.

In the following examples, we will translate expressions that contain words for the four fundamental operations of arithmetic, and we will always represent the unknown number by the letter x .

2.1 Examples

1.

A number increased by four equals ten.

Translation:

$$x + 4 = 10$$

2.

Five less than a number is eight

Translation:

$$x - 5 = 8$$

3.

The product of number and two equals sixteen

Translation:

$$2x = 16$$

4.

The quotient of a number and two is twenty

Translation:

$$\frac{x}{2} = 20$$

2.2 Equations and Expressions

Another concept the reader must understand is the difference between an equation and an expression. The distinction is that in an expression there is no equal sign. All of the above translations were equations. Below is an example of an expression:

$$5x^2 + xy - y^2$$

Notice, there is no equal sign!

2.3 Inequality Notation

A particular set of notations that you must understand for this course are the notations for equality and inequality. These symbols are used over and over again in advanced and fundamental mathematics. If we wish to write:

1.

Two equals two

In notation we write:

$$2 = 2$$

2.

Two does not equal five

In notation we write:

$$2 \neq 5$$

3.

Negative three is less than zero

$$-3 < 0$$

4.

A number less than or equal seven

In notation we write:

$$x \leq 7$$

Note that x could be replaced by $\{7, 6, 5, \dots\}$ and all the reals inbetween.

5.

Five is greater than negative four

In notation we write:

$$5 > -4$$

A number is greater than or equal to three

In notation we write:

$$x \geq 3$$

Note that x could be replaced by $\{3, 4, 5, \dots\}$ and all the reals inbetween.

To reiterate

$=$ means equal

\neq means not equal

$<$ means less than

\leq means less than or equal to

$>$ means greater than

\geq means greater than or equal to

2.4 Evaluation

In our last example, we show you how to evaluate a variable. The main idea is to replace the variable with a parentheses and then substitute in a value for that variable. In the example below, notice how we keep all the structures outside the variable the same after we replace the variable with a parentheses.

2.4.1 Example

Evaluate the given expression for $x = -2$

$$\begin{aligned} & \frac{-2x^2 - 3x}{x^3} \\ & \frac{-2(\)^2 - 3(\)}{(\)^3} \\ & \frac{-2(-2)^2 - 3(-2)}{(-2)^3} \\ & \frac{-2(4) + 6}{-8} \\ & \frac{-8 + 6}{-8} \\ & \frac{-2}{-8} \\ & \frac{1}{4} \end{aligned}$$

3 Facts

1. We can translate verbal statements about relationships between numbers into algebraic statements that describe the same relationships.
2. Relationships between number can be described by symbols for equality and inequality. Those symbols and their meanings are described below.

$=$ *means equal*

\neq means not equal

$<$ means less than

\leq means less than or equal to

$>$ means greater than

\geq means greater than or equal to

3. An algebraic equation contains an equal sign, an algebraic expression does not.
4. When evaluating algebraic expressions, we replace the variable(s) with parentheses, keeping all the structures outside the same, then we substitute in the value for the variable.

4 Exercises

1. Change the following to an algebraic expression, and use x to represent the unknown.

Five added to the product of a number and two.

2. Identify the following as an equation or an expression.

$$3x^3 - 57x + y$$

3. Fill in the blank with one of the following symbols to make the statement true: $=, \neq, <, \leq, >, \geq$.

$$5^2 \underline{\hspace{1cm}} 49$$

4. Determine if the following statement is true or false.

$$1 + \frac{2}{3} + 4\frac{1}{3} = 6$$

5. Evaluate the expression for the given value of the variable.

$$3m - 4m^2 + 1 ; m = \frac{1}{2}$$

5 Solutions

1.

$$5 + 2x$$

2. Expression

3.

$$25 \neq 49, \text{ or } 25 < 49, \text{ or } 25 \leq 49$$

4.

$$1 + \frac{2}{3} + 4\frac{1}{3} = 6$$

$$\frac{3}{3} + \frac{2}{3} + \frac{13}{3} = 6$$

$$\frac{18}{3} = 6$$

$$6 = 6$$

A true statement.

5.

$$3m - 4m^2 + 1 ; m = \frac{1}{2}$$

$$3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^2 + 1 ; m = \frac{1}{2}$$

$$\frac{3}{2} - \frac{4}{4} + 1 ; m = \frac{1}{2}$$

$$1\frac{1}{2} - 1 + 1$$

$$1\frac{1}{2}$$