1 Linear Equations in One Variable 2.2

At the end of this section, you will be able to solve the following problems:

1. $14x - 6 = -12$
2. $\frac{-3}{5}x = -15$
3. $7x + 8 = -8x + 2$
4. A number added to seven is equal to the difference of three times the number and seven.

2 Concepts

In a linear equation, no variable has a power greater that one. An example of a linear equation is:

$$3x + 1 = 7$$

When we look at this equation we see that some number, x, is multiplied by 3 and then one is added to it. Then, the result is set equal to seven. There is one and only one number that makes this statement true. The way we find that number is by reversing the order of operations to isolate the variable; we do all our additions and subtractions first and then our multiplications and divisions. Our goal is to get the term with x in it by itself on one side of the equation and then get x by itself. We begin by adding negative one to both sides. See below.

$$3x + 1 = 7 \tag{1}$$

$$\underline{\quad\quad -1 \;\; -1 \quad}$$

$$3x \quad = \quad 6 \tag{2}$$

In the next step, we divide both sides of the equation by 3.

$$\frac{3x}{3} = \frac{6}{3}$$

$$x = 2 \tag{3}$$

What we have done is use the laws of arithmetic to create a set of equivalent equations. If our arithmetic is right, we can write (1)=(2)=(3). So far, we have assumed that the laws of arithmetic will give us x, but there is no guarantee. To prove we have the right answer we must substitute the value we have found for x into the original equation to see if we have a true statement. The check is illustrated below:

$$3(2) + 1 = 7$$

$$7 = 7$$

It is obvious that the statement is true. Now we give another example of a linear equation and its solution.

2.1 Example

$$-2x + 5 = 4$$

$$\underline{\qquad -5 \;\; -5}$$

$$\frac{-2x}{-2} = \frac{-1}{-2}$$

$$x = \frac{-1}{-2}$$

$$x = \frac{1}{2} \; cancelling \; -1's \; in \; the \; numerator \; and \; denominator$$

Check:

$$-2(\frac{1}{2}) + 5 = 4$$

$$-1 + 5 = 4$$

$$4 = 4$$

Sometimes to solve a linear equation we must multiply both sides by a number to get x by itself.

2.2 Example

$$\frac{x}{3} = 4$$

$$(3)\frac{x}{3} = 4(3)$$

$$x = 12$$

When an equation has a fraction as a coefficient, that is not a unit fraction, it is useful to isolate the variable by multiplying both sides by the reciprocal of the coefficient.

2.3 Example

$$\frac{3x}{4} = \frac{7}{5}$$

$$(\frac{4}{3})\frac{3x}{4} = \frac{7}{5}(\frac{4}{3})$$

$$x = \frac{28}{15}$$

Notice that the goal is always to get the coefficient of x equal to 1.

Sometimes in a linear equation a variable is split on either side of the equation. For example,

$$3x - 4 = 7 + 5x$$

The goal here is to get all the variable on one side of the equation and all of the number on the other side of the equation. Look at our procedure below:

$$3x - 4 = 7 + 5x$$

$$\underline{-5x \qquad - 5x}$$

$$-2x - 4 = 7$$

$$\underline{+4 \quad + 4}$$

$$\frac{-2x}{-2} = \frac{11}{-2}$$

$$x = \frac{-11}{2}$$

Notice that the negative sign is in the numerator on the right hand side of the equation. The reason we can put the negative sign in the numerator is because we can change any two signs of a fraction without changing its value. For any ratio $\frac{a}{b}$ $b \neq 0$ We have:

$$+\frac{+a}{-b} = -\frac{+a}{+b} = +\frac{-a}{+b}$$

In practice, all you need to know is that if you have a negative sign in a denominator, you can put it in the numerator. The habit of making the numerator negative prevents dropping signs.

In application, we must be able to translate word statements into algebraic ones.

2.4 Example:

Four times a number increased by two is equal to seven.

The way we write four times a number is $4x$ and increased by means add. Putting it all together we have:

$$4x + 2 = 7$$

Another example would be:

Three times a number decreased by four is equal to the same number increased by one. This translates to be:

$$3x - 4 = x + 1$$

Below is a table of some common algebraic translations.

The quotient of a number and three	$\frac{x}{3}$
Two times the sum of a number and seven	$2(x+7)$
One-fifth of a number plus six	$\frac{1}{5}x+6$

2.5 Facts

1. When solving linear equations we use the order of operations in reverse. That is, we do all additions and subtractions first. Then we do all multiplication and divisions second etc.

2. A linear equation is one where all the variables in the equation have exponents equal to one.

3. To solve a linear equation we reduce the equation to a set of equivalent equations using the laws of arithmetic. The final goal is to get x by itself with a coefficient of positive one.

4. A linear equation in one variable has only one answer.

5. For any ratio $\frac{a}{b}$ $(b \neq 0)$ We have:

$$+\frac{+a}{-b} = -\frac{+a}{+b} = +\frac{-a}{+b}$$

3 Exercises

1.

$$14x - 6 = -12$$

2.

$$\frac{-3}{5}x = -15$$

3.

$$7x + 8 = -8x + 2$$

4. Solve the word problem: A number added to seven is equal to the difference of three times the number and seven.

4 Solutions

1.

$$14x - 6 = -12$$

$$\underline{\quad +6 \quad + 6 \quad}$$

$$\frac{14x}{14} = \frac{-6}{14}$$

$$x = \frac{-3}{7}$$

2.

$$\frac{-3}{5}x = -15$$

$$(\frac{5}{-3})\frac{-3}{5}x = -15(\frac{5}{-3})$$

$$x = 25$$

3.

$$7x + 8 = -8x + 2$$

$$\underline{+8x \qquad + 8x}$$

$$15x + 8 = 2$$

$$\underline{\quad -8 \qquad -8 \quad}$$

$$\frac{15x}{15} = \frac{-6}{15}$$

$$x = \frac{-2}{5}$$

4.

$$x + 7 = 3x - 7$$

$$\underline{\quad -7 \qquad\qquad -7 \quad}$$

$$x = 3x - 14$$

$$\underline{\quad -3x \;\; -3x \quad}$$

$$\frac{-2x}{-2} = \frac{-14}{-2}$$

$$x = 7$$