

1 More On Solving Linear Equations 2.3

By the end of this section, you will be able to solve the following problems.

1. Solve for x : $\frac{-3}{5} = \frac{4x}{3}$
2. Solve for x : $\frac{x}{9} + \frac{1}{12} = \frac{-2}{15}$
3. Solve for x : $7 - \frac{5}{6}x = 3 - \frac{2x}{7}$

2 Concepts

The distributive law says is that when a sum is multiplied by a constant, we can either multiply first and then add or add first and then multiply. The reason we can do this is because the answer will be the same. We give a numerical example below:

Multiplying first and then adding, we have:

$$4(2 - 3) = 4(2) + 4(-3) = 8 - 12 = -4$$

or adding signed numbers in parentheses first and then, multiplying, we have:

$$4(2 - 3) = 4(-1) = -4$$

In general, the distributive law is stated this way:

If a, b, c are any three real numbers then

$$a(b + c) = ab + ac$$

Now we solve an equation where we must use the distributive law. It is very important to remember when solving equations to:

1. Distributed first.
2. Collect like terms.
3. Get all the numbers on one side and all the variables on the other.

Now we solve an equation that makes use of the distributive law.

2.1 Example

$$3 - 4(x - 1) = 2x + 5$$

$$3 - 4x + 4 = 2x + 5$$

$$7 - 4x = 2x + 5$$

$$+4x \quad +4x$$

$$\overline{7 = 6x + 5}$$

$$-5 \quad -5$$

$$\overline{2 = 6x}$$

$$\frac{2}{6} = \frac{6x}{6}$$

$$\frac{1}{3} = x$$

Often, linear equations have fractions in them and it is important to learn the best approach to these. It is important to remember that when solving linear equations, we want to get rid of the denominators everywhere. We do this by multiplying both sides of the equation by the least common multiple of the denominators. That is, the smallest number that all the denominators fit into. Follow the example below.

$$3 + \frac{2}{3}(x - 1) = \frac{1}{2} + 5x$$

The least common multiple of 2 and 3 is 6, so we multiply every term by 6 on both sides of the equation.

$$(6)3 + (6)\frac{2}{3}(x - 1) = (6)\frac{1}{2} + (6)5x$$

$$18 + 4(x - 1) = 3 + 30x$$

$$18 + 4x - 4 = 3 + 30x$$

$$14 + 4x = 3 + 30x$$

$$-4x \quad -4x$$

$$\overline{14 = 3 + 26x}$$

$$-3 \quad -3$$

$$\overline{11 = 26x}$$

$$\frac{11}{26} = \frac{26x}{26}$$

$$\frac{11}{26} = x$$

2.2 Many Solutions and No Solution

Sometimes a linear equation does not have a solution and sometimes it has many solutions. We have already seen that many linear equation have just one solution. The following is an example of a linear equation that has no solution. Notice that the last equation is a contradiction.

$$7x - 4 = 3(x - 1) + 4(x + 2)$$

$$7x - 4 = 3x - 3 + 4x + 8$$

$$7x - 4 = 7x + 5$$

$$-7x \quad - 7x$$

$$\overline{-4 = 5}$$

When we reach a contradiction like the one above, we say the equation has no solution.

Another situation that we sometimes run into is when an equation has many solutions. This situation occurs when one side of the equation is the exact mirror of the other side. See the example below:

$$-(x + 5) + 2x - 2 = -3x - 4 + 4x - 3$$

$$-x - 5 + 2x - 2 = x - 7$$

$$x - 7 = x - 7$$

The last equation is a mirror image on both sides. That means is that any number could be substituted for x to make the statement true. Therefore, we say that the equation has many solutions.

3 Facts

1. The distributive law states that we may multiply first and then add or add first and then multiply and we write: For any real number a, b, c

$$a(b + c) = ab + ac$$

- Whenever we solve equations that contain denominators the first thing we want to do is get rid of all denominators by multiplying both sides by the LCM of all the denominators in the equation.
- Equations either have one solution, many solutions, or no solution.

4 Exercises

- Solve for x.

$$\frac{-3}{5} = \frac{4x}{3}$$

- Solve for x.

$$\frac{x}{9} + \frac{1}{12} = \frac{-2}{15}$$

- Solve for x.

$$7 - \frac{5}{6}x = 3 - \frac{2x}{7}$$

5 Solutions

1. Solve for x .

$$\frac{-3}{5} = \frac{4x}{3}$$

$$(15)\frac{-3}{5} = \frac{4x}{3}(15)$$

$$(3)(-3) = (4x)(5)$$

$$-9 = 20x$$

$$\frac{-9}{20} = \frac{20x}{20}$$

$$\frac{-9}{20} = x$$

2. Solve for x .

$$\frac{x}{9} + \frac{1}{12} = \frac{-2}{15}$$

$$(180)\frac{x}{9} + (180)\frac{1}{12} = (180)\frac{-2}{15}$$

$$20x + 15 = -24$$

$$-15 \quad -15$$

$$\overline{20x = -39}$$

$$\frac{20x}{20} = \frac{-39}{20}$$

$$x = \frac{-39}{20}$$

3. Solve for x .

$$7 - \frac{5}{6}x = 3 - \frac{2x}{7}$$

$$(42)7 - (42)\frac{5}{6}x = (42)3 - (42)\frac{2x}{7}$$

$$294 - 35x = 126 - 12x$$

$$-126 \quad -126$$

$$\overline{168 - 35x = -12x}$$

$$+35x \quad +35x$$

$$\frac{168}{23} = x$$