

# 1 Applications of Linear Equations 2.4

By the end of this section, you should be able to solve the following problems.

1. The length of a rectangular field is 4 meters more than two times its width. If the perimeter of the field is 32 meters, Find its dimensions.
2. Find three consecutive integers whose sum is 36.
3. In the 1960, United States Presidential Election, John F. Kennedy received 84 more electoral votes than Richard M. Nixon. Together they received 522 electoral votes. How many votes did each candidate receive?
4. The sum of the ages of mother and child is 40. If 4 years ago, the mothers age was 7 times the child's age, find how old is each one of them now.

Whenever we approach a word problem, there are several practical ideas to keep in mind. First and foremost, you have to understand the problem. Always ask yourself what is varying and what are the restrictions? Always try to write down what you know about the problem even if it is the most trivial thing, and always try to express what you know about the problem

in your own words. If you can draw a picture of what is going on in the problem, so much the better. Also, in the picture, try to introduce a variable or a number. Sometimes, it is helpful to ask yourself if there is a formula that will help you. We will now use these suggestions in the examples below.

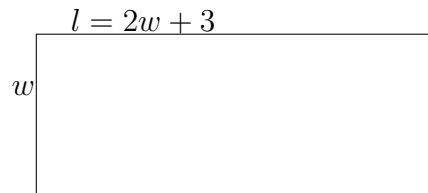
## 1.1 Example

A farmer wishes to enclose a rectangular field the length of which is two times the width plus 3 feet. What is the length and width of the fence if the farmer wants the perimeter of the field to be 600 feet?

We begin to solve the problem by identifying what is varying. Since the length of the field is expressed in terms of the width it seems like the width is varying. So a good *let step* would be:

*let  $w$  = the width of the field*

A next good step is draw a picture of what is going on in the problem. Since the length of the field is twice the width plus three, we draw the following diagram:



To solve this problem, we must use the perimeter formula:

$$P = 2l + 2w.$$

Since we have already expressed length in terms of width, we make the following substitutions:

$$600 = 2(2w + 3) + 2w$$

Notice that we have replaced  $l$  in the perimeter formula by  $2w + 3$  and we have replaced  $P$  by 600. Now we solve it.

$$600 = 2(2w + 3) + 2w$$

$$600 = 4w + 6 + 2w$$

$$600 = 6w + 6$$

$$-6 \quad -6$$

$$\overline{594 = 6w}$$

$$\frac{594}{6} = \frac{6w}{6}$$

$$99 = w$$

Now that we have found the width, all we have to do is substitute into the formula  $l = 2w + 3$  to get the length. So we have,

$$l = 2(99) + 3 = 201.$$

To make sure our solution checks, we substitute both the length and width into the perimeter equation:

$$600 = 2(201) + 2(99) = 402 + 198 = 600$$

and we see that the solution checks. In the next example, we examine a consecutive integer problem.

## 1.2 Example

The main thing to remember in a consecutive integer problem is that the way to get the next integer in a consecutive list is by adding one each time. So if the first number in a consecutive list is  $x$  then the next will be  $x + 1$  and the next will be  $x + 2$  and so on. Now we show how to solve a consecutive integer problem.

*The sum of three consecutive integers is 99. Find them.*

As our first step we make a *let step* and let  $x = \text{the first integer}$ . Therefore, the second integer is  $x + 1$  and the third is  $x + 2$ . Now we can write the equations as the sum of these integers equal to 99.

$$x + x + 1 + x + 2 = 99$$

$$3x + 3 = 99$$

$$\begin{array}{r} -3 \quad -3 \\ \hline \end{array}$$

$$\overline{3x = 96}$$

$$\frac{3x}{3} = \frac{96}{3}$$

$$x = 32$$

Therefore, the three consecutive integers are 32, 33, and 34. The sum is of course 99 as required.

### 1.3 Example

The following is an example of an age problem.

*6 years ago a father's age was 4 times his son's.*

*Now the sum of their ages is 72. How old is each?*

If we let the son's age equal  $x$  now we get the following table and equation.

	Then	Now
father	$4(x - 6)$	
son	$x - 6$	$x$

$$x - 6 + 4(x - 6) = 60$$

Notice that the sum must be 60 because the sum of the ages is six less for each. Now we proceed to solve.

$$x - 6 + 4(x - 6) = 60$$

$$x - 6 + 4x - 24 = 60$$

$$5x - 30 = 60$$

$$+30 \quad +30$$

$$\overline{5x = 90}$$

$$\frac{5x}{5} = \frac{90}{5}$$

$$x = 18$$

The result says that the sons's age is 18 now, so the fathers age then would be  $4((18) - 6) = 48$ . So the father's age now is  $48 + 6 = 54$ . This checks because  $18 + 54 = 72$ .

If we choose to let  $x$  represent the son's age then, we would have the following table and equation.

	Then	Now
father	$4x$	
son	$x$	$x + 6$

$$x + 4x = 60$$

Notice that the sum must be 60 because the sum of the ages is six less for each. Now we proceed to solve.

$$x + 4x = 60$$

$$5x = 60$$

$$\frac{5x}{5} = \frac{60}{5}$$

$$x = 12$$

This result says that the son's age then was 12. And since the father's age was four times that his age was 48. Therefore by adding six to both, we get  $18+54=72$ .

## 1.4 Example

In this example, we add two numbers together that differ by a fixed amount.

*Together Mary and Elizabeth sell 384 lawn chairs.*

*If Elizabeth sells 54 more lawn chairs than does Mary,*

*how many does each one sell?*

Here we let

*$x =$  number lawn chairs that Mary sells*

Therefore, Elizabeth sells  $x + 54$ . Adding the two together we have:

$$x + x + 54 = 384$$

$$2x + 54 = 384$$

$$\begin{array}{r} -54 \\ -54 \end{array}$$

$$\hline 2x = 330$$

$$\frac{2x}{2} = \frac{330}{2}$$

$$x = 165$$

Therefore, Mary sells 165 lawn chairs and Elizabeth sells 219 lawn chairs.

## 2 Facts

1. When solving word problems, always write down what is known or given.
2. When solving word problems, decide what is varying and make a *let step*.
3. When solving word problems, draw a picture to help you understand the problem. Label your diagram with variables, numbers or both.
4. Always check your results against the original conditions of the problem.

## 3 Exercises

1. The length of a rectangular field is 4 meters more than two times its width. If the perimeter of the field is 32 meters, Find its dimensions.
2. Find three consecutive integers whose sum is 36.
3. In the 1960 United States Presidential Election, John F. Kennedy received 84 more electoral votes than Richard M. Nixon. Together they

received 522 electoral votes. How many votes did each candidate receive?

4. The sum of the ages of mother and child is 40. If 4 years ago, the mothers age was 7 times the child's age, find how old is each one of them now.

## 4 Solutions

1. let  $w =$  *width of the rectangle*. Then we can express the length as

$l = 2w + 4$  Our equation will be:

$$32 = 2(2w + 4) + 2w$$

$$32 = 4w + 8 + 2w$$

$$32 = 6w + 8$$

$$\begin{array}{r} -8 \\ -8 \end{array}$$

$$\overline{24 = 6w}$$

$$\frac{24}{6} = \frac{6w}{6}$$

$$4 = w$$

If the width is 4, then the length is 12

2. let  $x =$  *the first consecutive integer*

so  $x + 1$  will be the second and  $x + 2$  will be the third. So our equation

will be:

$$x + x + 1 + x + 2 = 36$$

$$\overline{3x + 3 = 36}$$

$$\begin{array}{r} -3 \quad -3 \\ \hline \end{array}$$

$$\overline{3x = 33}$$

$$\frac{3x}{3} = \frac{33}{3}$$

$$x = 11$$

So the three consecutive integers in question are 11, 12, 13.

3. Let  $x =$  *the number of Nixon's electoral votes*. Then  $x + 84$  is the number for Kennedy. Therefore,

$$x + x + 84 = 522$$

$$2x + 84 = 522$$

$$\begin{array}{r} -84 \quad -84 \\ \hline \end{array}$$

$$\overline{2x = 438}$$

$$\frac{2x}{2} = \frac{438}{2}$$

$$x = 219$$

So Nixon got 219 electoral votes and Kennedy got 303.

4. Let  $x =$  *the child's age four years ago*. Then  $7x$  is the age of the mother four years ago. The sum will be less eight because mother and

child were both for years younger.

$$x + 7x = 32$$

$$8x = 32$$

$$\frac{8x}{8} = \frac{32}{8}$$

$$x = 4$$

If the child was 4 years old four years ago, the mother was 28. Therefore, the child is 8 now and the mother is 32.