

# 1 More Applications 2.7

By the end of this section, you should be able to solve the following problems:

1. 20 liters of pure acid are mixed with a solution having 70% acid to have a new solution with 80% acid. How many liters of solution with 70% acid are needed?
2. An automobile dealer has \$600,000 in compact cars and mid-size cars. The profit on the sale of the compact cars is 1%, and profit on the sale of mid-size cars is 16%. How much did the dealer invest in mid-size cars if the profit on the total investment is 12%.
3. Hesse wishes to earn \$3500 per year in supplemental income from an inheritance of \$40,000. He puts \$10,000 in a bank paying 8%. What rate must he earn on the remainder so that his annual income is \$3500?
4. Two cyclists start out at the same time towards each other from point 150 miles apart. After how many hours will they cross each other, if the first and second cyclist travel respectively at speeds of 40 and 50 miles per hour.

## 2 Concepts

For now, before we get to simultaneous linear equations, we must learn to express one quantity in terms of the other. Our first example is a mixture problem.

### 2.1 Example

A merchant wishes to add a mixture that is 40% French Roast coffee beans to 80 pounds of pure Colombian coffee beans to produce a mixture that is 70% Colombian coffee. How many pounds of French Roast must be added to produce such a mixture? For a minute let's forget about the percentages. For now, all we know that we are going to add 80 pounds of Columbian coffee beans  $x$  pounds of French Roast beans to produce  $x + 80$  coffee beans. We know that the Columbian beans are pure and that French Roast beans are only 40% pure and we need to get a 70% mixture. So, multiplying by the correct percentages, we have the following equation:

$$100\%(80) + 40\%x = 70\%(80 + x)$$

$$80 + .4x = .7(80) + .7x$$

$$80 + .4x = 56 + .7x$$

$$-56 \quad - .4x \quad -56 \quad - .4x$$

$$\overline{24 = .3x}$$

$$\frac{24}{.3} = \frac{.3x}{.3}$$

$$80 = x$$

This means we need 80 pounds of the French Roast.

Checking:

$$80 + .4(80) = 56 + .7(80)$$

$$80 + 32 = 56 + 56$$

$$112 = 112$$

In this next problem, there is dollar amount that is invested at two different interest levels so we must express the two quantities as  $x$  and the dollar amount minus  $x$ .

## 2.2 Example

An investor has \$4000 dollars to invest. He wishes to invest part of his money at 8% interest and part at 12% interest so his yield is 10% on his original

\$4000 investment. Exactly how much should he invest at 8% to meet this requirement.

In this problem, our let step should be: Let  $x$  = the amount invested at 8%. And the way we will represent the amount invested at 12% will be  $4000-x$ . Without any percentages the equation will be

$$x + 4000 - x = 4000$$

Now we identify the amounts by their percentages.

$$8\%x + 12\%(4000 - x) = 10\%(4000)$$

rewriting we have

$$.08x + .12(4000 - x) = .10(4000)$$

$$.08x + 480 - .12x = 400$$

$$-.04x + 480 = 400$$

$$-480 \quad - 480$$

$$\hline -.04x = -80$$

$$\frac{-.04x}{-.04} = \frac{-80}{-.04}$$

$$x = 2000$$

In our next problem, we use the idea that  $rate \times time = distance$ .

### 2.3 Example

Two automobiles leave a parking lot at the same time. The first automobile travels at 60mph and the second travels at 50mph. At what time will the two automobiles be 220 miles apart?

For our first step we say: *let  $t =$  the time when the two automobiles are 220 miles apart.*

Because  $rate \times time = distance$ , the distance the first car travels is  $60t$  and the distance the second car travels is  $50t$ . Adding these two distances together we have

$$60t + 50t = 220$$

$$110t = 220$$

$$\frac{110t}{110} = \frac{220}{110}$$

$$t = 2$$

So the cars will be 220 miles apart in 2 hours.

### 3 Facts

1. Any quantity  $a$  can be separated into two parts by writing:  $x + a - x$
2. Any percentage can be converted into a decimal number by moving the decimal point two places to the left.
3. Distance=Rate $\times$ time

### 4 Exercises

1. 20 liters of pure acid are mixed with a solution having 70% acid to have a new solution with 80% acid. How many liters of solution with 70% acid are needed?
2. An automobile dealer has \$600,000 in compact cars and mid-sized cars. The profit on the sale of compact cars is 1%, and the profit on the sale of mid-sized cars is 16%. How much did the dealer invest in mid-sized cars if the profit on the overall total investment is 12%.
3. Hesse wishes to earn \$3500 per year in supplemental income from an inheritance of \$40,000. He puts \$10,000 in a bank paying 8%. What rate must he earn on the remainder so that his annual income is \$3500?

- Two cyclist start out at the same time towards each other from points 150 miles apart. After how many hours will they cross each other, if the first and second cyclist travel respectively at speeds of 40 and 50 miles per hour.

## 5 Solutions

1.

$$100\%(20) + 70\%x = 80\%(20 + x)$$

$$20 + .70x = 16 + .80x$$

$$-16 \quad - .70x \quad -16 \quad - .70x$$

$$\overline{4 = .10x}$$

$$\frac{4}{.10} = \frac{.10x}{.10}$$

$$40 = x$$

2. The profit plus the initial investment on compact cars is  $100\% + 1\% = 101\%$ . The profit plus the initial investment on mid-size cars is  $100\% + 16\% = 116\%$ .

$$101\%x + 116\%(600,000 - x) = 112\%(600,000)$$

$$1.01x + 696000 - 1.16x = 672000$$

$$-.15x + 696000 = 672000$$

$$-696000 \quad -696000$$

$$\overline{-.15x = -24000}$$

$$\frac{-.15x}{-.15} = \frac{-24000}{-.15}$$

$$x = 160,000$$

3.

$$8\%(10,000) + x\%(30,000) = 3500$$

$$800 + 300x = 3500$$

$$-800 \quad -800$$

$$\overline{300x = 2700}$$

$$\frac{300x}{300} = \frac{2700}{300}$$

$$x = 9\%$$

4. Let  $t$  = the time both cyclists travel. Then  $40t$  = the distance the first cyclist travel and  $50t$  = the distance the second cyclist travels. Then,

$$40t + 50t = 150$$

$$90t = 150$$

$$\frac{90t}{90} = \frac{150}{90}$$

$$t = 1\frac{2}{3}$$