

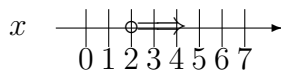
# 1 Solutions of Linear Inequalities 2.8

By the end of this section, you should be able to solve the following problems:

1. Solve and graph the solutions set.  $5 - (2 - z) > 2z + 5$
2. Solve and graph the solution set.  $5(2k - 3) \geq -9 + 4(k + 2) + 4$
3. Solve for  $x$ .  $6(x + 1) \geq 5 + (x + 3) - 3x$
4. Solve for  $x$ .  $\frac{1}{2}(5x - 7) \leq \frac{-1}{4}(6 - 2x)$

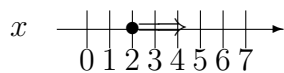
## 2 Concepts

Whenever we express an equation with inequalities in it, the solution will describe a range of values. For example,  $x > 2$  means all the values that  $x$  can assume are strictly greater than 2. Notice that we allow values that are as close to 2 as we wish so long as it is not 2 itself. For example 2.0000001 is o.k. because it is greater than 2. We would graph the relation  $x > 2$  as below:

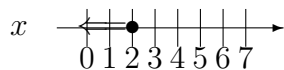


Notice the arrow indicating the set points to the right. Also notice the hole on 2 to show it is not included.

Another relation we want to describe is  $x \geq 2$ . What this means is  $x$  can take on any value greater than or equal to 2. In the graph below, we fill in the hole on 2 to show that the point is included.



And if we wanted to show the graph of  $x \leq 2$ , the arrow for the set would point to the left.



Now we go on to solve algebraic statements with inequalities in them. The main thing to remember here is that whenever we multiply or divide by a negative number, we must reverse the direction of the inequality. For example, suppose we multiply  $2 < 3$  on both sides by  $-1$ . Observe:

$$2 < 3$$

$$(-1)2 < (-1)3$$

$$-2 < -3$$

But this statement doesn't make any sense because -2 is not less than -3. The way we get out of this jam is by reversing the direction of the inequality

$$-2 > -3$$

Therefore, whenever we multiply or divide by a negative number, we reverse the direction of the inequality. In our next example, we solve an inequality and graph its solution set.

## 2.1 Example

Solve the inequalities and graph its solution set.

$$5x - (7 + x) \geq 4 + 3x$$

$$5x - 7 - x \geq 4 + 3x$$

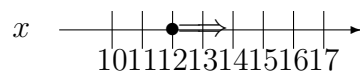
$$4x - 7 \geq 4 + 3x$$

$$-3x \quad -3x$$

$$\overline{x - 7 \geq 4}$$

$$+7 \quad + 7$$

$$\overline{x \geq 11}$$



In the next problem, we simply solve the inequality without graphing the solution set.

## 2.2 Example

Solve the equation.

$$\frac{1}{2}(3x - 2) \leq \frac{1}{3}(5x + 1)$$

$$(6)\frac{1}{2}(3x - 2) \leq (6)\frac{1}{3}(5x + 1)$$

$$3(3x - 2) \leq 2(5x + 1)$$

$$9x - 6 \leq 10x + 2$$

$$-10x \quad - 10x$$

$$\overline{-x - 6 \leq 2}$$

$$+6 \quad +6$$

$$\overline{-x \leq 8}$$

Change the direction of the inequality after dividing through by negative one.

$$x \geq -8$$

### 3 Facts

1.  $>$  is read, "Strictly greater than"
2.  $<$  is read, "Strictly less than"
3.  $\geq$  is read, "Greater than or equal to"
4.  $\leq$  is read, "Less than or equal to"
5. Whenever we multiply or divide by a negative number, we reverse the direction of the inequality.

## 4 Exercises

1. Solve and graph the solution set

$$5 - (2 - z) > 2z + 5$$

2. Solve and graph the solution set

$$5(2k - 3) \geq -9 + 4(k + 2) + 4$$

3. Solve:

$$6(x + 1) \geq 5 + (x + 3) - 3x$$

4. Solve:

$$\frac{1}{2}(5x - 7) \leq \frac{-1}{4}(6 - 2x)$$

## 5 Solutions

1. Solve and graph the solution set

$$5 - (2 - z) > 2z + 5$$

$$5 - 2 + z > 2z + 5$$

$$3 + z > 2z + 5$$

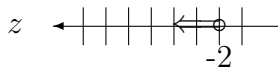
$$-2z \quad -2z$$

$$\overline{3 - z > 5}$$

$$-3 \quad -3$$

$$\overline{-z > 2}$$

$$z < -2$$



2. Solve and graph the solution set

$$5(2k - 3) \geq -9 + 4(k + 2) + 4$$

$$10k - 15 \geq -9 + 4k + 8 + 4$$

$$10k - 15 \geq 3 + 4k$$

$$-4k \quad -4k$$

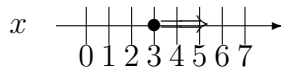
$$\overline{6k - 15 \geq 3}$$

$$+15 \quad +15$$

$$\overline{6k \geq 18}$$

$$\frac{6k}{6} \geq \frac{18}{6}$$

$$k \geq 3$$



3. Solve:

$$6(x + 1) \geq 5 + (x + 3) - 3x$$

$$6x + 6 \geq 5 + x + 3 - 3x$$

$$6x + 6 \geq 8 - 2x$$

$$2x \quad 2x$$

$$8x + 6 \geq 8$$

$$-6 \quad -6$$

$$8x \geq 2$$

$$\frac{8x}{8} \geq \frac{2}{8}$$

$$x \geq \frac{1}{4}$$

4. Solve:

$$\frac{1}{2}(5x - 7) \leq \frac{-1}{4}(6 - 2x)$$

$$(4)\frac{1}{2}(5x - 7) \leq (4)\frac{-1}{4}(6 - 2x)$$

$$2(5x - 7) \leq -1(6 - 2x)$$

$$10x - 14 \leq -6 + 2x$$

$$-2x \quad -2x$$

$$\overline{8x - 14 \leq -6}$$

$$+14 \quad +14$$

$$\overline{8x \leq 8}$$

$$\frac{8x}{8} \leq \frac{8}{8}$$

$$x \leq \frac{1}{1}$$