

1 Linear Equations in Two Variables 3.1

By the end of this section, you should be able to solve the following problems.

1. Complete the ordered pair so that it represents a solution to the given equation.

$$(0, ?); \quad 3x + 5y = 10$$

2. Complete the set ordered pairs so that each ordered pair is a solution to the given equation.

$$\{(-2, ?), (0, ?), (5, ?)\}$$

$$y + 3x = 4x$$

3. Complete the set of ordered pairs so that each ordered pair is a solution to the given equation.

$$\{(-2, ?), (?, 3), (2, ?)\}$$

$$3x + 2y = 6$$

4. Without plotting the point, determine the quadrant in which the point lies. (a)(-4,3) (b)(4,-3) (c)(-4,-3) (d)(4,3)

2 Concepts

An equation in two variables is like any other equation—some values will make the statement true and some values will not. When we are looking for a pair of numbers that make a sentence in two variables true we fix one variable and search for the other. In our first example, we will fix the x value and use algebra to find the y value. In other examples, we will fix the y value and use algebra to find the x .

2.1 Example

If $x = 2$ in the equation

$$3x + 2y = 12$$

What does y equal. First we substitute in for x .

$$3(2) + 2y = 12$$

$$6 + 2y = 12$$

$$-6 \quad -6$$

$$\overline{2y = 6}$$

$$\frac{2y}{2} = \frac{6}{2}$$

$$y = 3$$

What we have found is the ordered pair (2,3) that satisfies $3x + 2y = 12$. An ordered pair is two number written in parentheses where the x -coordinate is given first and the y -coordinate is given second. In our next example, we fix values for both x and y alternately and find the value that completes the ordered pair and solves the equation.

2.2 Example

For $\{(-2, ?), (?, 4), (-1, ?)\}$ Find the missing coordinates that makes the following statement true.

$$-2x + 3y = 6.$$

For $(-2, ?)$ we substitute -2 in for x :

$$-2(-2) + 3y = 6$$

$$4 + 3y = 6$$

$$-4 \quad -4$$

$$3y = 2$$

$$\frac{3y}{3} = \frac{2}{3}$$

$$y = \frac{2}{3}$$

For $(?,4)$ we substitute 4 in for y .

$$-2x + 3(4) = 6$$

$$-2x + 12 = 6$$

$$-12 \quad -12$$

$$\underline{-2x = 6}$$

$$\frac{-2x}{-2} = \frac{-6}{-2}$$

$$x = 3$$

For $(-1,?)$, we substitute -1 in for x .

$$-2(-1) + 3y = 6$$

$$2 + 3y = 6$$

$$-2 \quad -2$$

$$3y = 4$$

$$\frac{3y}{3} = \frac{4}{3}$$

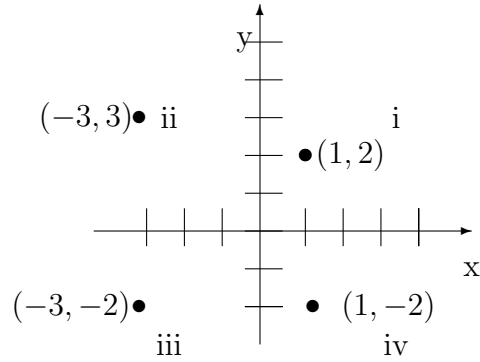
$$y = \frac{4}{3}$$

What we have found is the following set of ordered pairs that satisfy the equation.

$$\left\{ \left(-2, \frac{2}{3}\right), (3, 4), \left(-1, \frac{4}{3}\right) \right\}$$

3 Concepts

It is important to know where points are located just by the signs of the coordinates. The signs of coordinates in xy -plane depend on the quadrant they are in. All the points in the first quadrant have a positive value for x and y . All the points in the second quadrant have a negative x value and a positive y value. All the points in the third quadrant have a negative x value and a negative y value, and in the fourth quadrant all the x values have a positive and the y values are negative. Below is an illustration of the quadrants, their signs and some sample points.



3.1 Example

1. $(3,4)$ is in quadrant I because both coordinates are positive
2. $(-2,-3)$ is in quadrant III because both coordinates are negative
3. $(5,-7)$ is in quadrant IV because the first coordinate is positive and the second coordinate is negative
4. $(-2,3)$ is in Quadrant II because the first coordinate is negative and the second coordinate is positive

4 Facts

1. To find the missing coordinate in an equation with two variables, we substitute in for either x or y and solve to find the other coordinate.
2. All points in Quadrant I have the sign pattern $(+,+)$
3. All points in Quadrant II have the sign pattern $(-,+)$
4. All points in Quadrant III have the sign pattern $(-,-)$
5. All points in Quadrant IV have the sign pattern $(+,-)$

5 Exercises

1. Complete the ordered pair so that it represents a solution to the given equation:

$$(0, ?); \quad 3x + 5y = 10$$

2. Complete the set of ordered pairs so that each ordered pair is a solution to the equation.

$$\{(-2, ?), (0, ?), (5, ?)\}$$

$$y + 3x = 4x$$

3. Complete the set of ordered pairs so that each ordered pair is a solution to the given equation

$$\{(-3, ?), (?, 3), (2, ?)\}$$

$$3x + 2y = 6$$

4. Without plotting the point, determine the quadrant in which the point lies. (a)(-4,3) (b)(4,-3) (c)(-4,-3) (d)(4,3)

6 Solutions

1. Complete the ordered pair so that it represents a solution to the given equation

$$(0, ?); \quad 3x + 5y = 10$$

$$3(0) + 5y = 10$$

$$5y = 10$$

$$\frac{5y}{5} = \frac{10}{5}$$

$$y = 2$$

Therefore, $(0, 2)$ is an ordered pair that makes the equation true.

2. Complete the set of ordered pairs so that each ordered pair is a solution to the equation.

$$\{(-2, ?), (0, ?), (5, ?)\}$$

$$y + 3x = 4x$$

Substituting -2 in for x we have:

$$y - 3(-2) = 4$$

$$y - 6 = 4$$

$$+6 \quad +6$$

$$\overline{y = 10}$$

Substituting in 0 for x we have:

$$y + 3(0) = 4$$

$$y = 4$$

Substituting in 5 for x we have:

$$y - 3(5) = 4$$

$$y - 15 = 4$$

$$+15 \quad +15$$

$$\overline{y = 19}$$

Therefore, for the three given values the solution set is:

$$\{(-2, 10), (0, 4), (5, 19)\}$$

3. Complete the set of ordered pairs so that each ordered pair is a solution to the given equation

$$\{(-3, ?), (?, 3), (2, ?)\}$$

$$3x + 2y = 6$$

For $x=-2$ we have,

$$3(-2) + 2y = 6$$

$$-6 + 2y = 6$$

$$+6 \quad +6$$

$$\overline{2y = 12}$$

$$\frac{2y}{2} = \frac{12}{2}$$

$$y = 6$$

For $y=3$ we have,

$$3x + 2(3) = 6$$

$$3x + 6 = 6$$

$$-6 \quad -6$$

$$\overline{3x = 0}$$

$$\frac{3x}{3} = \frac{0}{3}$$

$$x = 0$$

For $x=2$ we have,

$$3(2) + 2y = 6$$

$$6 + 2y = 6$$

$$-6 \quad -6$$

$$\overline{2y = 0}$$

$$\frac{2y}{2} = \frac{0}{2}$$

$$y = 0$$

Therefore, the solution set is

$$\{(-2, 6), (0, 3), (2, 0)\}$$

4. Without plotting the point, determine the quadrant in which the point

lies. (a)(-4,3) (b)(4,-3) (c)(-4,-3) (d)(4,3)

(a)ii (b)iv (c)iii (d)i