

1 Slope of a Line 3.3

By the end of this section, you should be able to solve the following problems.

1. Find the slope of a line passing through the given pair of points.

$$(4, 1) \text{ and } (-5, 1)$$

2. Without drawing the graph of the line, determine whether the line is rising or falling from left to right.

$$2x + 3y = 9$$

3. Determine whether the given pair of lines is parallel, perpendicular, or neither.

$$2x + 3y = 4$$

$$6y = 5 - 4x$$

4. Determine whether the given pair of lines is parallel, perpendicular, or neither.

$$2x + 3y = 9$$

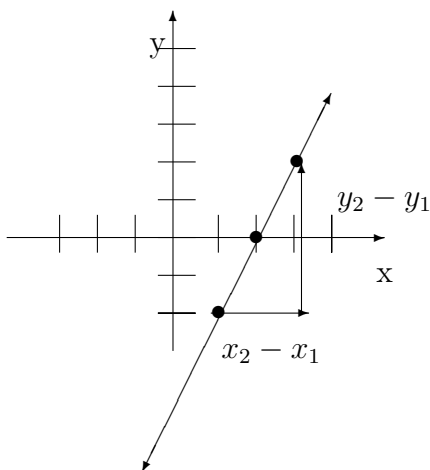
$$4x + 6y = 13$$

2 Concepts

A slope is a rate of change. In this course we discuss the rate of change of a line between two points on the line. Specifically we form the ratio of the difference between the y coordinates and the difference between the x coordinates of two points. It is customary to use the letter m for the slope and analytically we write:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Geometrically, we think of the slope as the ratio between the vertical change between two points and the horizontal change between two points on a line.



In our first example, we find the slope of a line passing through two points.

2.1 Example

Find the slope of the line passing through the points

$$(1, -2) \text{ and } (4, 5)$$

First we label the points carefully before plugging into the equation.

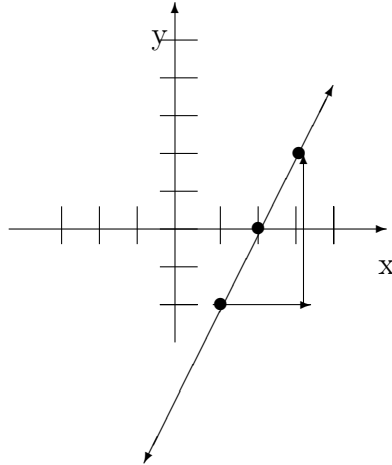
$$(x_1 = 1, y_1 = -2) \text{ and } (x_2 = 4, y_2 = 5)$$

Substituting we have:

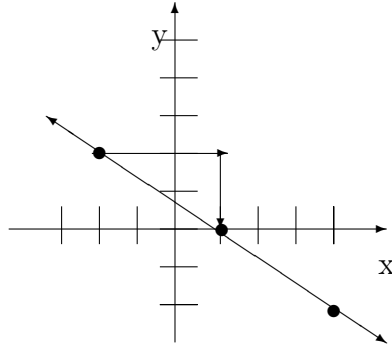
$$\begin{aligned} m &= \frac{(5) - (-2)}{(4) - (1)} \\ &= \frac{7}{3} \end{aligned}$$

The sign of the slope gives us special information about how the line behaves.

If the slope is positive, the rises from right to left. In the graph below the heads of the vectors in the right triangle below the line both point in the positive direction, so their ratio will be positive.



In the next graph, one vector head, in the right triangle, is pointing in the positive direction and the other is pointing in the negative direction, so their ratio will be negative.



The *standard form* for the equation of a line is

$$Ax + By = C.$$

Usually the coefficient of x is written as a positive number. Note that if we had an equation with a negative coefficient for x , we could multiply both side by -1 to make the coefficient positive. For example,

$$-2x + 3y = 5,$$

after multiplying both sides by -1 , the above becomes

$$2x - 3y = -5.$$

Another form of the equation of a line is called *slope-intercept form*. This is written as

$$y = mx + b.$$

In this equation, m is the slope and b is the y -coordinate where the line crosses the y -axis. Any equation of a line can always be written in slope-intercept form. For example,

$$\begin{aligned} 3x - 4y &= 12 \\ -3x &\quad - 3x \\ \hline -4y &= -3x + 12 \\ \frac{-4y}{-4} &= \frac{-3x + 12}{-4} \\ y &= \frac{3x}{4} - 3 \end{aligned}$$

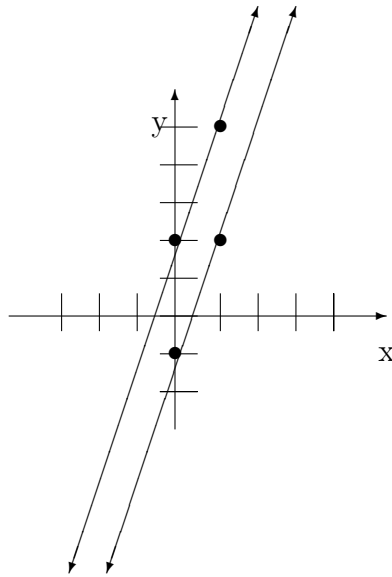
This form of the equation is useful to us because we can see the slope coefficient at a glance. Recognizing that it is a positive $\frac{3}{4}$, we know that the line is rising to the right.

The following lines are parallel because they have the same slope.

$$y = 3x + 2$$

$$y = 3x - 1$$

See the graph below.

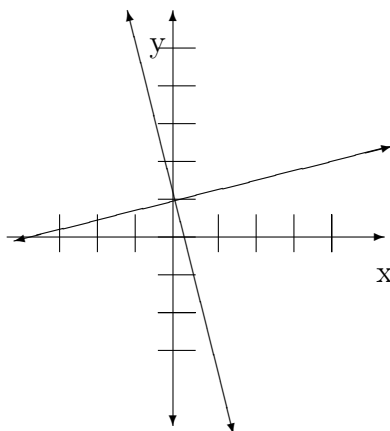


In fact, all lines that have the same slope are parallel. Let's examine the graphs of

$$y = -4x + 1$$

and

$$y = \frac{1}{4}x + 1.$$



Notice that the graphs are perpendicular. In fact, whenever the slopes of two lines are negative reciprocals of one another, the lines are perpendicular.

We apply these principles to the next example.

Determine if the following equations are parallel, perpendicular, or neither.

$$3x - 2y = 6$$

$$5x + 3y = 4$$

First, rewrite the equations in $y = mx + b$ form so we can see the slopes.

Rewriting the first equation:

$$3x - 2y = 6$$

$$\begin{aligned}
 & -3x \quad - 3x \\
 & \hline
 & -2y = -3x + 6 \\
 & \frac{-2y}{-2} = \frac{-3x + 6}{-2} \\
 & y = \frac{3x}{2} - 3
 \end{aligned}$$

Rewriting the second equation.

$$\begin{aligned}
 & 5x + 3y = 4 \\
 & -5x \quad - 5x \\
 & \hline
 & 3y = -5x + 4 \\
 & \frac{3y}{3} = \frac{-5x + 4}{3} \\
 & y = \frac{-5x}{3} + \frac{4}{3}
 \end{aligned}$$

The slope of the first equation is $m_1 = \frac{3}{2}$ and the $m_2 = \frac{-5}{3}$. Since the slopes are not the same, nor are they negative reciprocals of one another, these lines neither perpendicular or parallel, which means that they intersect at a point but not at right angles.

3 Facts

1. The equation for the slope of a line is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

2. The standard form of the equation of a line is:

$$Ax + By = C.$$

In standard form, if A is negative, the equation is rewritten so that A is positive.

3. The slope-intercept form of an equation is:

$$y = mx + b.$$

Where m is the slope and b is the y -coordinate of the point of intersection of the line and the y -axis.

4. Parallel lines have the same slope, and perpendicular lines have slopes that are negative reciprocals of one another.

4 Exercises

1. Find the slope of a line passing through the given pair of points.

$$(4, 1) \text{ and } (-5, 1)$$

2. Without drawing the graph of the line, determine whether the line is rising or falling from left to right.

$$2x + 3y = 9$$

3. Determine whether the given pair of lines is parallel, perpendicular, or neither.

$$2x + 3y = 4$$

$$6y = 5 - 4x$$

4. Determine whether the given pair of lines is parallel, perpendicular, or neither.

$$2x + 3y = 9$$

$$4x + 6y = 13$$

5 Solutions

1. Find the slope of a line passing through the given pair of points.

$$(4, 1) \text{ and } (-5, 1)$$

First label the points carefully:

$$(x_1 = 4, y_1 = 1) \text{ and } (x_2 = -5, y_2 = 1)$$

Substituting into the slope equation we have:

$$m = \frac{(1) - (1)}{(-5) - (4)}$$

$$m = \frac{0}{-9} = 0$$

2. Without drawing the graph of the line, determine whether the line is rising or falling from left to right.

$$2x + 3y = 9$$

$$-2x \quad -2x$$

$$\overline{3y = 2x + 9}$$

$$\frac{3y}{3} = \frac{-2x + 9}{3}$$

$$y = \frac{-2x}{3} + 3$$

Since the slope is negative, we know that the slope is falling from left to right.

3. Determine whether the given pair of lines is parallel, perpendicular, or neither.

$$2x + 3y = 4$$

$$6y = 5 - 4x$$

Converting to $y = mx + b$ form.

$$2x + 3y = 4$$

$$-2x \quad -2x$$

$$\overline{3y = -2x + 4}$$

$$\frac{3y}{3} = \frac{-2x + 4}{3}$$

$$y = \frac{-2x}{3} + \frac{4}{3}$$

$$\frac{6y}{6} = \frac{5 - 4x}{6}$$

$$y = \frac{5}{6} - \frac{2x}{3}$$

Since the slope are the same, the lines are parallel.

4. Determine whether the given pair of lines is parallel, perpendicular, or neither.

$$2x + 3y = 9$$

$$4x + 6y = 13$$

Converting to $y = mx + b$ form.

$$2x + 3y = 9$$

$$-2x \quad -2x$$

$$\overline{3y = -2x + 9}$$

$$\frac{3y}{3} = \frac{-2x + 9}{3}$$

$$y = \frac{-2x}{3} + 3$$

$$4x + 6y = 13$$

$$-4x \quad -4x$$

$$\overline{6y = -4x + 13}$$

$$\frac{6y}{6} = \frac{-4x + 13}{6}$$

$$y = \frac{-2x}{3} + \frac{13}{6}$$

Since the slopes are the same, the lines are parallel.