1 Functions and Relations 3.6

By the end of this section, you should be able to solve the following problems.

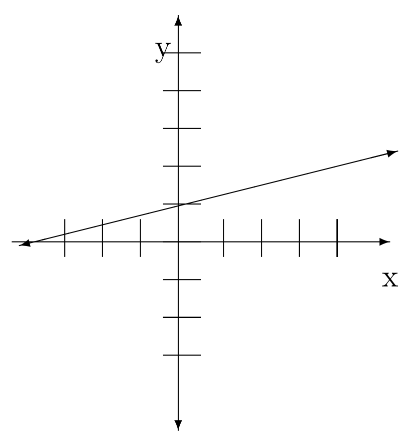
1. Determine if the relation is a function? Indicate the domain and range.

$$\{(1,5),(1,6),(2,5),(2,7),(3,6),(4,7)\}$$

2. Determine if the relations is a function? Indicate the domain and range.

$$\{(2,3),(3,4),(4,5),(5,6)\}$$

3. Identify whether or not the relation defined by the graph is a function.

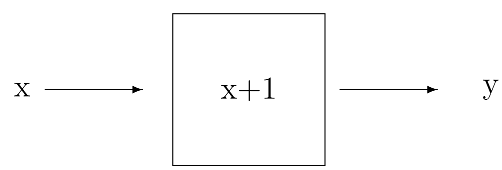


4. Evaluate the function at the given values.

$$f(x) = -3x^2 + 5x; f(4), f(-4)$$

2 Concepts

A function is like a stupid machine. For instance, a function may add 1 to all the input values. Below, the box that has $x + 1$ written on it will take every value that we designate as x and increase it by 1. The y values are the results after the "machine" has done its job.



We have special names for the set of x values that we put into the machine and the y values that come out. We call the $x's$ the *domain* values and the $y's$ the *range* values. We now write the definition of a function.

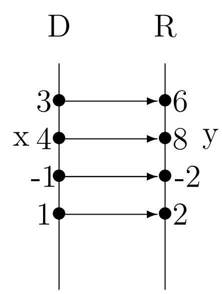
Definition: A function is a set of ordered pairs (x, y) no two of which have the same first member.

2.1 Example

Determine if the following set of pairings is a function.

$$\{(3,6),(4,8),(-1,-2),(1,2)\}$$

We can analyze this set by using a "stick" diagram to show exactly how the elements of the domain and range match up. On the left hand "stick" we list each distinct element of the domain only once, and on the left hand "stick" we list each element of the range only once.



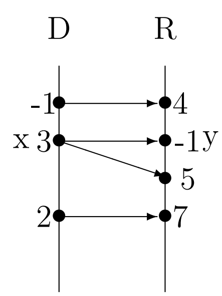
In the diagram, we see that each distinct value in the domain is paired with one and only one value in the range. Not all pairings are like this as we see in the next example.

2.2 Example

Determine whether or not the following set of pairing describes a function.

$$\{(-1,4),(3,-1),(3,5),(2,7)\}$$

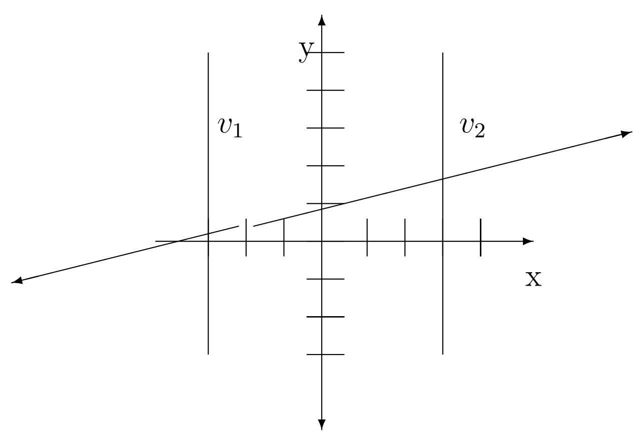
Now we analyze the stick diagram making sure that when we draw it we never repeat values.



In this diagram, each distinct value in the domain does not get sent to one and only one y-value in the range because the element 3 in the domain is sent to both 5 and -1. Therefore, this set of pairings does not describe a function. If a set of pairs does not describe a function, the best we can say is that it describes a relation. **Definition** A relation is a set of ordered pairs (x, y).

3 Concepts

Another way to examine the concept of a function is with what is called the vertical line test. If the graph of a relation is a function, then, when a vertical line is passed through it, it will never pass through more than one point at a time. For example, in the graph below if we pass a vertical line through it at any point; no more than one point will be intersected.



Functions also have specific notations that we should be familiar with. The notation f(x) pronounced, "f of x" is often use designate the function idea. The notation, f(x), has nothing to do with multiplying f by x. The x in this case is called the argument of the function and whatever number is placed in the argument is substituted for x everywhere on the other side of the equation. We illustrate below.

3.1 Example

Evaluate the function at the given values for x.

$$f(x) = \frac{1}{2}x^2 + 5x - 1$$

Find:

1. f(4)
2. f(-2)

1.

$$f(4) = \frac{1}{2}(4)^2 + 5(4) - 1 =$$

$$f(4) = \frac{1}{2}(4)^2 + 5(4) - 1$$

$$\frac{1}{2}(16) + 20 - 1$$

$$8 + 20 - 1$$

2.

$$f(-2) = \frac{1}{2}(-2)^2 + 5(-2) - 1$$

$$f(-2) = \frac{1}{2}(4) - 10 - 1$$

$$f(-2) = 2 - 10 - 1$$

$$f(-2) = -9$$

4 Facts

1. A function is a set of pairings such that each distinct element in the domain is assigned to one and only one element in the range.

2. Every function is a relation, but not all relations are functions.

3. If a graph is the graph of a function, then a vertical line will only intersect one point on the graph everywhere on the graph.

4. The notation f(x) is read, “f of x” and it means put whatever number is replaced for x in for x on the other side of the equation.

5 Exercises

1. Determine which of the relations is a function?

   Indicate the domain and range.

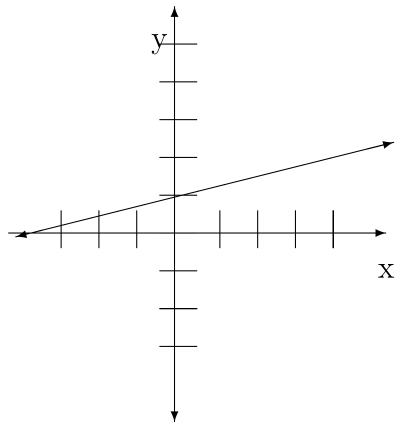
$$\{(2,3),(3,4),(4,5)\}$$

2. Determine if the relation is a function?

   Indicate the domain and range.

$$\{(1,5),(1,6),(2,7)\}$$

3. Identify whether or not the relation defined by the graph is a function.



4. Find the indicated value.

$$f(x) = 2 - 3x^2 + 5x;\ \ f(4), f(-4)$$

6 Solutions

1. Determine which of the relations is a function?

   Indicate the domain and range.

$$\{(2,3),(3,4),(4,5)\}$$

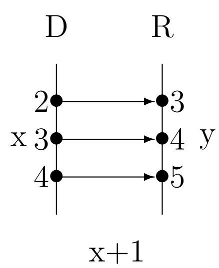
The domain of the function is:

$$\{2,3,4\}$$

The range of the function is:

$$\{3,4,5\}$$

In this set each distinct element in the domain is sent to one and only one element in the range.



2. Determine if the relation is a function?

   Indicate the domain and range.

$$\{(1,5),(1,6),(2,7)\}$$

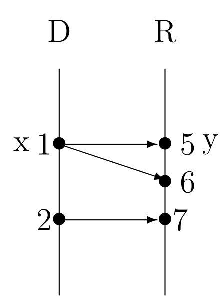
   The domain set is:

$$\{1,2\}$$

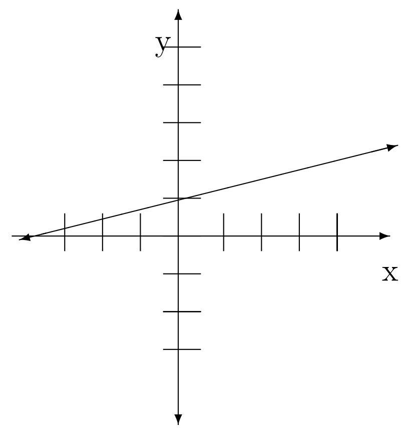
   The range set is:

$$\{5,6,7\}$$

   This set of pairings does not describe a function because the domain element 1 is "sent" to both 5 and 6.



3. Identify whether or not the relation defined by the graph is a function.



No vertical line will intersect this graph in more than one point at a time. Therefore, the graph describes a function.

4. Find the indicated value.

$$f(x)2 - 3x^2 + 5x;\ f(4), f(-4)$$

$$f(4) = -3(4)^2 + 5(4)$$

$$f(4) = -3(16) + 20$$

$$f(4) = -48 + 20$$

$$f(4) = -28$$

$$f(-4) = -3(-4)^2 + 5(-4)$$

$$f(-4) = -3(16) - 20$$

$$f(-4) = -48 - 20$$

$$f(-4) = -68$$