

1 Solving Using Graphs 4.1

By the end of this section, you should be able to solve the following problems.

1. Identify if the ordered pair $(0,1)$ is a solution of the given system.

$$3y = x + 18$$

$$x + y = 6$$

2. Identify if the indicated ordered pair is $(-2,-3)$ a solution of the given system.

$$3x + 2y = 0$$

$$2x = 3y - 5$$

3. Solve the system of equations by graphing. Identify whether the lines are intersecting, parallel, or coincident.

$$3x = 5 - 2y$$

$$3x + 2y = 7$$

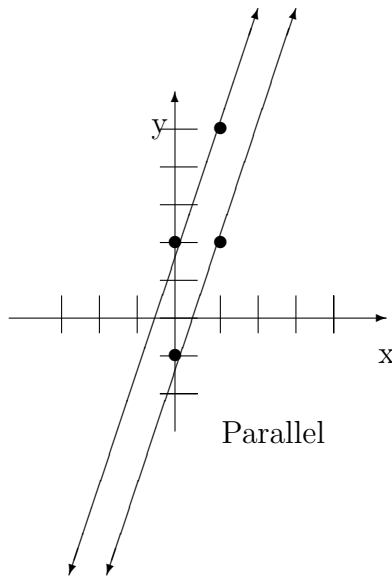
4. Express each equation in slope-intercept form. Determine whether the lines are parallel, intersecting, or coincident.

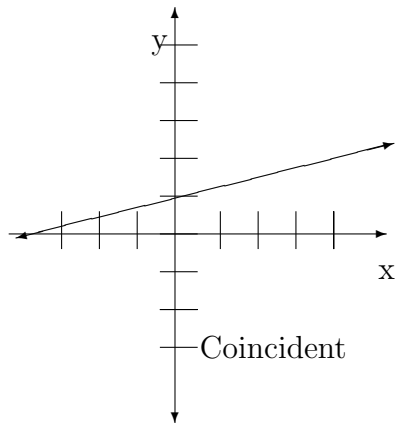
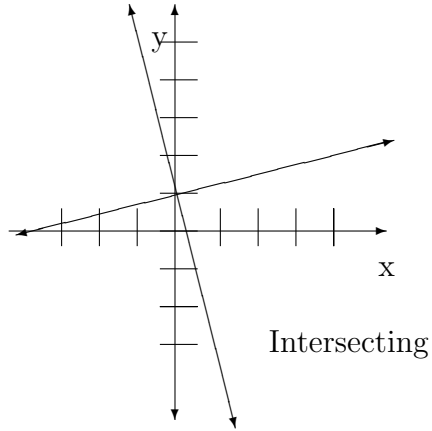
$$3x - 4y = 12$$

$$6x - 8y = 48$$

2 Concepts

Whenever we graph a system of two equations in two variables equations in the xy -plane there are only 3 possible outcomes. The first is that the two lines intersect; the second is that the two lines are parallel, and the third possibilities is that the graphs of the equation intersect everywhere. That is, one graph is virtually on top of the other. When the graphs are on top of one another, we say they are coincident graphs. The graphs below show the three possibilities.





In the following example we examine whether or not the system intersects, is parallel, or coincident. Then we graph the system and check the result.

2.1 Example

For the following system, use $y = mx + b$ form to identify whether the system of equations is coincident, intersecting, or parallel. Graph the two equations on the same in the xy-plane to check your result.

$$-2x + y = 1$$

$$-x + y = -1$$

Converting each equation to $y = mx + b$ form we have:

$$-2x + y = 1$$

$$2x \quad 2x$$

$$\overline{y = 2x + 1}$$

$$-x + y = -1$$

$$x \quad x$$

$$\overline{y = x - 1}$$

In the first equation, we see that the slope is 2. In the second equation, we see that the slope is 1. Because the slopes are not the same, we know that the lines are not parallel. Because the equations are not identical we know that the lines are not coincident. Therefore, the lines must intersect.

For

$$y = 2x + 1$$

we plot

$$\{(0, 1), (1, 3), (2, 5)\}$$

because

$$y = 2(0) + 1 = 0 + 1 = 1$$

$$y = 2(1) + 1 = 2 + 1 = 3$$

$$y = 2(2) + 1 = 4 + 1 = 5$$

For

$$y = x - 1$$

We plot

$$\{(1, 0), (2, 1), (3, 2)\}$$

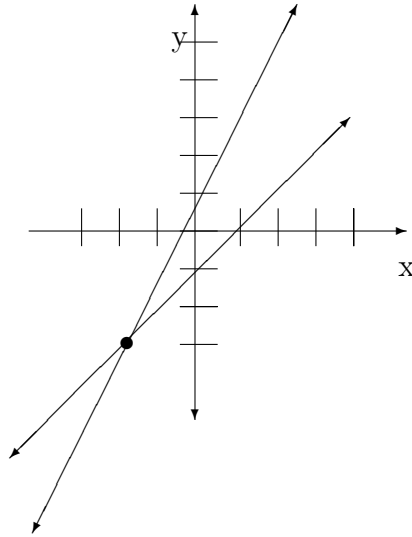
because

$$y = (1) - 1 = 0$$

$$y = (2) - 1 = 1$$

$$y = (3) - 1 = 2$$

From the graph, we see that the point where the two lines intersect is
(-2,-3)



Now we check the solution in both equations to make sure we are right.

For

$$-2x + y = 1$$

we get

$$-2(-2) + (-3) = 1$$

$$4 - 3 = 1$$

a true statement.

For

$$-x + y = -1$$

we get

$$-(-2) + (-3) = -1$$

$$2 + (-3) = -1$$

a true statement.

Since the solution solves *both* equations, we know it is the solution to the system.

3 Facts

1. Two lines in the plane either intersect, are parallel, or coincident.
2. Any solution point to a system of equations must satisfy both equations.
3. To see if two lines are parallel, put both into $y=mx+b$ form and check to see if their slopes are equal.
4. To see if two lines are coincident, put both into $y=mx+b$ form and check to see if the equations are exactly the same. Note: you must reduce all fractions in $y = mx + b$ form.
- 5.

4 Exercises

1. Identify if the ordered pair $(0,1)$ is a solution of the given system.

$$3y = x + 18$$

$$x + y = 6$$

2. Identify if the indicated ordered pair is $(-2,-3)$ a solution of the given system.

$$3x + 2y = 0$$

$$2x = 3y - 5$$

3. Solve the system of equations by graphing, and identify whether the lines are intersecting, parallel, or coincident.

$$3x = 5 - 2y$$

$$3x + 2y = 7$$

4. Express each equation in slope-intercept form. Determine whether the lines are parallel, intersecting, or coincident.

$$3x - 4y = 12$$

$$6x - 8y = 48$$

5 Solutions

1. Identify if the ordered pair $(0,1)$ is a solution of the given system.

$$3y = x + 18$$

$$x + y = 6$$

For

$$3y = x + 18,$$

we have

$$3(1) = (0) + 18$$

$$3 = 18$$

This is not a true statement; therefore, $(0,1)$ is not a solution to the system.

2. Identify if the indicated ordered pair is $(-2,-3)$ a solution of the given system.

$$3x + 2y = 0$$

$$2x = 3y - 5$$

For

$$3x + 2y = 0,$$

We have

$$3(-2) + 2(-3) - 0$$

$$-12 = 0$$

This is not a true statement; therefore, $(-2,-3)$ is not a solution.

3. Solve the system of equations by graphing, and identify whether the lines are intersecting, parallel, or coincident.

$$3x = 5 - 2y$$

$$3x + 2y = 7$$

In $y = mx + b$ form

$$3x = 5 - 2y$$

$$-5 \quad -5$$

$$-2y = 3x - 5$$

$$\frac{-2y}{-2} = \frac{3x - 5}{-2}$$

$$y = \frac{3x - 5}{-2}$$

$$y = \frac{-3}{2}x + \frac{5}{2}$$

For

$$3x + 2y = 7$$

we have

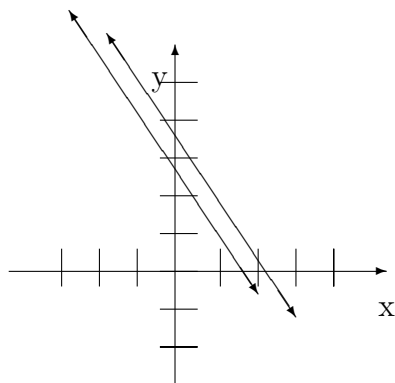
$$-3x \quad -3x$$

$$2y = -3x + 7$$

$$\frac{2y}{2} = \frac{-3x + 7}{2}$$

$$y = \frac{-3}{2}x + \frac{7}{2}$$

The system has no solution. The lines are parallel. See the graph below.



4. Express each equation in slope-intercept form. Determine whether the lines are parallel, intersecting, or coincident.

$$3x - 4y = 12$$

$$6x - 8y = 48$$

For

$$3x - 4y = 12,$$

$$-3x \quad -3x$$

$$\overline{-4y = -3x + 12}$$

$$\frac{-4y}{-4} = \frac{-3x + 12}{-4}$$

$$y = \frac{3}{4}x - 3$$

For

$$6x - 8y = 48,$$

$$-6x \quad -6x$$

$$\overline{-8y = -6x + 48}$$

$$\frac{-8y}{-8} = \frac{-6x + 48}{-8}$$

$$y = \frac{3}{4}x - 6$$

Since the lines have the same slope, they are parallel.