1 Solving By Substitution 4.2

By the end of this section, you should be able to solve the following problems.

1. Solve the system of equations using the substitution method.

$$5x + 3y = 6$$

$$x = 3y + 5$$

2. Solve the system of equations using the substitution method.

$$x = 4 - y$$

$$4x + 4y = 16$$

3. Solve the system of equations using substitution method. Using the solution, identify whether the graph of the system consists of intersecting, parallel, or coincident lines.

$$x + y = 3$$

$$2x - y = 12$$

4. Solve the system of equations using substitution method. Using the solution, identify whether the graph of the system consists of intersecting,

parallel, or coincident lines.

$$2x = 10y + 14$$

$$6x - 2y = 2x + 28$$

2 Concepts

When we use the substitution method to solve equations, all we are doing is eliminating a variable in one equation so that we can solve for y or x. The idea is to solve for x or y in one equation and then substitute for that variable into the other equation. An example will illustrate.

2.1 Example

Solve the system of equations using substitution.

$$(A) \ -2x + y = 1$$

$$(B) \ -x + y = -1$$

We may solve for x or y in either equation, by it is simpler to solve for y

because it has a coefficient of 1. We solve for y in equation A.

$$-2x + y = 1$$

$$2x \quad 2x$$

$$\overline{y = 2x + 1}$$

Now we substitute in for y in equation B.

$$-x + (2x + 1) = -1$$

$$-x + 2x + 1 = -1$$

$$x + 1 = -1$$

$$-1 \quad -1$$

$$\overline{x = -2}$$

Substituting for x back into the original equation for A.

$$-2(-2) + y = 1$$

$$4 + y = 1$$

$$-4 \quad -4$$

$$\overline{y = -3}$$

We must check the solution in both equations to see that the solution is correct.

$$A \ -2(-2)+(-3)=1$$

$$4-3=1$$

$$B \ -(-2)+(-3)=-1$$

$$2-3=-1$$

2.2 Example

Use the substitution method to solve the following equation. Use the solution to identify the equations as parallel, coincident, or intersecting.

$$A \ 3x-6y=12$$

$$B \ 2x-4y=-8$$

We wish to solve for x using equation A. To get a coefficient of 1 for x, we divide the entire equation by 3.

$$\frac{1}{3}(3x-6y=12)$$

$$x-2y=4$$

Solving for x we get.

$$+2y \quad +2y$$

$$\overline{x = 2y + 4}$$

Substituting into equation B we get:

$$2(2y + 4) - 4y = -8$$

$$4y + 8 - 4y = -8$$

$$8 = -8$$

This false statement tells us that the equations are parallel.

3 Facts

1. When using substitution, we isolate the variable that has a coefficient of 1.

2. When the system of equations has a solution, we must test to se that it solves both equations.

3. If after substituting for a variable and solving, we get an illogical statement like: $2 = 3$, we Know that the lines are parallel.

4. If after substituting and solving, one side of the equation is the mirror image of the other like: $2x + 1 = 2x + 1$, then we know the lines are coincident

4 Exercises

1. Solve the system of equations using substitution method.

$$5x + 3y = 6$$

$$x = 3y + 5$$

2. Solve the system of equations using substitution method.

$$x = 4 - y$$

$$4x + 4y = 16$$

3. Solve the system of equations using substitution method. Using the solution, identify whether the graph of the system consists of intersecting, parallel, or coincident line.

$$x + y = 3$$

$$2x - y = 12$$

4. Solve the system of equations using substitution method. Using the solution, identify whether the graph of the system consists of intersecting, parallel, or coincident lines.

$$2x = 10y + 14$$

$$6x - 2y = 2x + 28$$

5 Solutions

1. Solve the system of equations using substitution method.

$$5x + 3y = 6$$

$$x = 3y + 5$$

$$5(3y + 5) + 3y = 6$$

$$15y + 25 + 3y = 6$$

$$18y + 25 = 6$$

$$-25 \quad -25$$

$$\overline{18y = -19}$$

$$\frac{18y}{18} = \frac{-19}{18}$$

$$y = \frac{-19}{18}$$

$$x = 3(\frac{-19}{18}) + 5$$

$$x = \frac{-19}{6} + 5$$

$$x = \frac{-19}{6} + 5\frac{6}{6}$$

$$x = \frac{-19}{6} + \frac{30}{6}$$

$$x = \frac{11}{6}$$

2. Solve the system of equations using substitution method.

$$x = 4 - y$$

$$4x + 4y = 16$$

$$4(4 - y) + 4y = 18$$

$$16 - 4y + 4y = 16$$

$$16 = 16$$

These equations are coincident.

3. Solve the system of equations using substitution method. Using the solution, identify whether the graph of the system consists of intersecting, parallel, or coincident line.

$$A\ x + y = 3$$

$$B\ 2x - y = 12$$

$$A\ x + y = 3$$

$$-y \quad -y$$

$$\overline{x = 3 - y}$$

$$2(3 - y) - y = 12$$

$$6 - 2y - y = 12$$

$$6 - 3y = 12$$

$$-6 \quad -6$$

$$\overline{-3y = 6}$$

$$\frac{-3y}{-3} = \frac{6}{-3}$$

$$A\ x + (-2) = 3$$

$$2 \quad 2$$

$$\overline{x = 5}$$

The graphs of these equations intersect at (5,-2)

4. Solve the system of equations using substitution method. Using the solution, identify whether the graph of the system consists of intersecting, parallel, or coincident lines.

$$A\ 2x = 10y + 14$$

B

$$B\ 6x - 2y = 2x + 28$$

$$A\ \frac{2x}{2} = \frac{10y + 14}{2}$$

10

$$x = 5y + 7$$

$$B\ 6x - 2y = 2x + 28$$

$$-2x \quad -2x$$

$$\overline{4x - 2y = 28}$$

$$4(5y + 7) - 2y = 28$$

$$20y + 28 - 2y = 28$$

$$18y + 28 = 28$$

$$-28 \quad -28$$

$$\overline{18y = 0}$$

$$\frac{18y}{18} = \frac{0}{18}$$

$$y = 0$$

$$2A\ x = 10(0) + 14$$

$$2x = 14$$

$$\frac{2x}{2} = \frac{14}{2}$$

$$x = 7$$

The graphs of these equations intersect at (7,0).