1 Solving Using Elimination By Addition 4.3

By the end of this section, you should be able to solve the following problems.

1. Solve the system of equations using elimination by addition.

$$2x + 3y = 8$$

$$3x - 2y = -1$$

2. Solve the system of equating using elimination by addition.

$$2x - y = 7$$

$$4x + 3y = 9$$

3. Solve the system of equations using elimination by addition. Using the solution identify whether the graph of the system consists of intersecting, parallel or coincident lines.

$$2x - 3y = 7$$

$$x + y = 1$$

4. Solve the system of equations using elimination by addition. Using the solution identify whether the graph of the system consists of intersect-

ing, parallel, or coincident lines.

$$\frac{x}{3} + \frac{y}{3} = 4$$

$$\frac{x}{4} - \frac{y}{4} - 1$$

2 Concepts

The main goal, when we see two equations with two unknowns, is to eliminate one of the variables. In the previous section, we did this by substitution, now we do it by elimination. When we eliminate a variable, the goal is to make the coefficients of the variables the same in absolute value but opposite in sign. Once this is done, we simply add the two equations together to eliminate one variable, then solve for the remaining variable. An example will illustrate.

2.1 Example

$$A\ 3x - 2y = 6$$

$$B\ 2x + 4y = 8$$

The easiest course of action here is eliminate the $y's$ because they are opposite in sign, so all that remains to be done is to multiply equation (A) by 2.

$$A\ 2(3x - 2y = 6) \Rightarrow 6x - 4y = 12$$

$$6x - 4y = 12$$

$$2x + 4y = 8$$

By addition we get.

$$\overline{8x = 20}$$

$$\frac{8x}{8} = \frac{20}{8}$$

$$x = \frac{5}{2}$$

Substituting into equation (B) we solve for y.

$$B2(\frac{5}{2}) + 4y = 8$$

$$5 + 4y = 8$$

$$-5 \quad -5$$

$$\overline{4y = 3}$$

$$\frac{4y}{4} = \frac{3}{4}$$

$$y = \frac{3}{4}$$

Always check your result in both equations. Check:

$$3(\frac{5}{2}) - 2(\frac{3}{4}) = 6$$

$$\frac{15}{2} - \frac{3}{4} = \frac{12}{2} = 6$$

$$2(\frac{5}{2}) + 4(\frac{3}{4}) = 8$$

$$5 + 3 = 8$$

In our next example, we see that sometimes we have to multiply both equations by a constant to eliminate one variable.

2.2 Example

$$(A)2x - 5y = 2$$

$$(B)3x + 4y = 5$$

In this system of equations, we again choose to eliminate the $y's$, but to do that we must multiply equation (A) by 4 and (B) by 5. See below.

$$4(2x - 5y = 2) \Rightarrow 8x - 20y = 8$$

$$5(3x + 4y = 5) \Rightarrow 15x + 20y = 25$$

$$8x - 20y = 8$$

$$15x + 20y = 25$$

By addition we get.

$$\overline{23x = 33}$$

$$\frac{23x}{23} = \frac{33}{23}$$

$$x = \frac{33}{23}$$

We now substitute back into one of the original equations to find y. We chose equation (A).

$$2(\frac{33}{23}) - 5y = 2$$

$$\frac{66}{23} - 5y = 2$$

$$-\frac{66}{23} \quad -\frac{66}{23}$$

$$\overline{-5y = 2 - \frac{66}{23}}$$

$$-5y = \frac{46}{23} - \frac{66}{23}$$

$$-5y = \frac{-20}{23}$$

$$\frac{-5y}{-5} = \frac{\frac{-20}{23}}{-5}$$

$$y = \frac{4}{23}$$

3 Facts

1. We may multipy one or both equations by a constant to make a variable drop out.

2. When finding the other variable, make sure to substitute back into one of the original equations.

3. We may choose to eliminate either variable, the solution to the system will be the same.

4 Exercises

1. Solve the system of equation using elimination by addition.

$$2x + 3y = 8$$

$$3x - 2y = -8$$

2. Solve the system of equations using elimination by addition.

$$2x - y = 7$$

$$4x + 3y = 9$$

3. Solve the system of equations using elimination by addition. Using the solution, identify whether the graph of the system consists of intersecting, parallel or coincident lines

$$2x - 3y = 7$$

$$x + y = 1$$

4. Solve the system of equations using elimination by addition. Using the solution, identify whether the graph of the system consists of intersecting, parallel, or coincident lines.

$$\frac{x}{3} + \frac{y}{3} = 4$$

$$\frac{x}{4} + \frac{y}{4} = -1$$

5 Solutions

1. Solve the system of equation using elimination by addition.

$$2x + 3y = 8$$

$$3x - 2y = -8$$

$$2(2x + 3y = 8) \Rightarrow 4x + 6y = 16$$

$$3(3x - 2y = -1) \Rightarrow 9x - 6y = -3$$

$$4x + 6y = 16$$

$$9x - 6y = -3$$

$$\overline{13x = 13}$$

$$\frac{13x}{13} = \frac{13}{13}$$

$$x = 1$$

$$2(1) + 3y = 8$$

$$2 + 3y = 8$$

$$-2 \quad -2$$

$$\overline{3y = 6}$$

$$\frac{3y}{3} = \frac{6}{3}$$

$$8$$

$$y = 2$$

2. Solve the system of equations using elimination by addition.

$$2x - y = 7$$

$$4x + 3y = 9$$

$$3(2x - y = 7) \Rightarrow 6x - 3y = 21$$

$$6x - 3y = 21$$

$$4x + 3y = 9$$

$$\overline{10x = 30}$$

$$\frac{10x}{10} = \frac{30}{10}$$

$$x = 3$$

$$2(3) - y = 7$$

$$6 - y = 7$$

$$-6 \quad -6$$

$$\overline{-y = 1}$$

$$y = -1$$

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3. Solve the system of equations using elimination by addition. Using the solution, identify whether the graph of the system consists of intersecting, parallel, or coincident lines

$$2x - 3y = 7$$

$$x + y = 1$$

$$3(x + y = 1) \Rightarrow 3x + 3y = 3$$

$$2x - 3y = 7$$

$$3x + 3y = 3$$

$$\overline{5x = 10}$$

$$\frac{5x}{5} = \frac{10}{5}$$

$$x = 2$$

$$2(2) - 3y = 7$$

$$4 - 3y = 7$$

$$-4 \quad -4$$

$$-3y = 3$$

$$\frac{-3y}{-3} = \frac{3}{-3}$$

$$10$$

$$y = -1$$

The system intersects at (2,-1)

4. Solve the system of equations using elimination by addition. Using the solution, identify whether the graph of the system consists of intersecting, parallel, or coincident lines.

$$\frac{x}{3} + \frac{y}{3} = 4$$

$$\frac{x}{4} + \frac{y}{4} = -1$$

$$\frac{x+y}{3} = 4$$

$$\frac{x+y}{4} = -1$$

$$x + y = 12$$

$$x + y = -4$$

$$-1(x + y = 12) \Rightarrow -x - y = -12$$

$$-x - y = -12$$

$$x + y = -4$$

$$\overline{0 = -16}$$

This is a parallel system of equations. There is no solution.