

1 Applications 4.5

By the end of this section, you should be able to solve the following problems.

1. Rollins invested part of \$6000 at 10% and the rest at 8%. His annual income from these investments is \$556. How much did he invest at each rate?
2. A lab technician has 15% and 40% concentrations of nitric acid. A class is conducting an experiment that requires 300 milliliters of a 25% concentration of nitric acid. How many milliliters of each concentration should be mixed to make the desired concentration?
3. An airplane can fly 360 miles into the wind in 3 hours. If the plane reverses the direction and the wind condition remain the same, the return trip takes only two hours. Find the speed of the plane in still air.
4. A number consists of two digits whose sum is 5. When the digits are reversed, the sum is increased by 9. Find the number.

2 Concepts

There is a pattern to problems that involve two equations and two unknowns. In coin and mixture problems, there is an equation that describes the total amount of material and another that describes specific rates or values. Consider the following example.

2.1 Example

A jar contains 46 coins consisting of nickels and quarters only. The total amount of money in the jar is \$5.50. How many nickels and how many quarters does the jar contain?

We begin with our ‘Let’ steps.

Let x = the number of nickels

Let y = the number of quarters.

We can model the total number coins without regard to value by:

$$(A) \quad x + y = 46$$

We introduce monetary value by introducing the fraction of a dollar each

coin is worth. We call this our *value equation*.

$$(B) .05x + .25y = 5.50$$

After multiplying equation B by 100 to get rid of the decimal point and equation A by -5 we have:

$$-5x - 5y = -230$$

$$5x + 25y = 550$$

$$\overline{20y = 320}$$

$$\frac{20y}{20} = \frac{320}{20}$$

$$y = 16$$

Substituting:

$$x + (16) = 46$$

$$-16 \quad -16$$

$$\overline{x = 30}$$

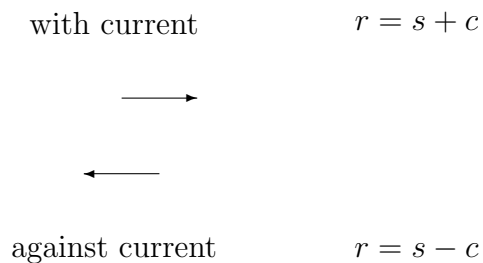
$$Check : (30) + (16) = 46$$

$$\frac{3}{2} + 4 = \frac{11}{2}$$

In our next problems we solve a rate times time equals distance problem ($rt = d$). Usually, in these problems wind or current is a variable that is subtracted from rate of the plane or boat. We will explain with an example.

2.2 Example

A boat can travel 12 miles up stream against the current in 2 hours and down stream with it in half the time. Find the speed of the boat in still water and the speed of the current? First we draw a picture and introduce notation.



We can model this situation using our $rt = d$ equation, but we must include the current.

Let the rate against the current be equal to the speed of boat minus the current: $s - c$. So let the rate with the current be equal to $s + c$.

Now we introduce the time. The trip up stream can be modelled by:

$$A \quad 2(s - c) = 12$$

and since the trip down stream takes half as long we have:

$$B \quad s + c = 12$$

to eliminate c we multiply B by 2, so we have:

$$2s - 2c = 12$$

$$2s + 2c = 24$$

$$\hline 4s = 36$$

$$\frac{4s}{4} = \frac{36}{4}$$

$$s = 9$$

$$\text{Check : } 2[(9) - (3)] = 12$$

$$(9) + (3) = 12$$

Therefore,

$$c = 3.$$

The speed of the boat in still water is 9 miles per hour and the speed of the current is 3 miles per hour.

3 Facts

1. Whenever possible, draw a picture of the problem situation and put in a usable notation.
2. Always identify variables when solving word problems with two equations and two unknowns.
3. The relationship $d = rt$ is often used in problems involving two equations and two unknowns.
4. Visualize what is going on in the problem and model the situation to the best of your ability using algebraic equations.
5. Always check your results.

4 Exercises

1. Rollins invested part of \$6000 at 10% and the rest at 8%. His annual income from these investments is \$556. How much did he invest at each rate?
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5 Solutions

1. Rollins invested part of \$6000 at 10% and the rest at 8%. His annual income from these investments is \$556. How much did he invest at each rate?

$$x + y = 6000$$

$$.10x + .08y = \$556$$

$$-10x - 10y = -60000$$

$$10x + 8y = 55600$$

$$\overline{-2y = -4400}$$

$$\frac{-2y}{-2} = \frac{-4400}{-2}$$

$$y = 2200$$

$$x = 3800$$

2. A lab technician has 15% and 40% concentrations of nitric acid. A class is conducting an experiment that requires 300 milliliters of a 25%

concentration of nitric acid. How many milliliters of each concentration should be mixed to make the desired concentration?

$$x + y = 300$$

$$.15x + .40y = .25(300)$$

$$-15x - 15y = -4500$$

$$15x + 40y = 7500$$

$$\overline{25y = 3000}$$

$$\frac{25y}{25} = \frac{3000}{25}$$

$$y = 120$$

$$x = 180$$

3. An airplane can fly 360 miles into the wind in 3 hours. If the plane reverses the direction and the wind conditions remain the same, the return trip takes only 2 hours. Find the speed of the plane in still air.

$$3(s - w) = 360$$

$$2(s + w) = 360$$

$$3s - 3w = 360$$

$$2s + 2w = 360$$

$$6s - 6w = 720$$

$$6s + 6w = 1080$$

$$12s = 1800$$

$$\frac{12s}{12} = \frac{1800}{12}$$

$$s = 150$$

$$w = 30$$

4. A number consists of two digits whose sum is 5. When the digits are reversed, the number is increased by 9. Find the number.

Let $x =$ the first digit

and

$y =$ the second digit

Then

$10x + y =$ the number

and

$$10y + x = \text{the number with the digits reversed.}$$

We know that:

$$A \quad x + y = 5$$

and

$$B \quad 10y + x = 10x + y + 9$$

Simplifying B we have:

$$9y - 9x = 9$$

dividing both sides by 9 we have

$$-x + y = 1$$

$$x + y = 5$$

$$\overline{2y = 6}$$

$$y = 3$$

$$x = 2$$

So our number is 23.