1 Exponents 5.1

By the end of the section, you should be able to solve the following problems.

1. Simplify the expression using the power rule.

$$(-4x^2y)^3$$

2. Simplify the expression using the power rule.

$$(\frac{-2x^2y}{5z})^2$$

3. Simplify

$$-2xy(x^2y)^3 - 5x^5(xy^2)^2$$

4. Simplify

$$3x^3(3x^2y)^2 - 4x(x^3y)^2$$

2 Concepts

In this section, we learn how to multiply exponential expressions, how to raise exponential expressions to powers, and how to raise products and quotients to powers. But first we have to know the parts of an exponential expression. In the monomial x^3, x is called the base and 3 is called the exponent, and

$x^3 = x \cdot x \cdot x$. In the expression $3x^2$, it is essential that you understand that only x is being raised to the power 2. Again, in the expression $-x^5$ only x is being raised to the fifth power not the understood -1 that is begin multiplied by x^5. If we look at $x^2 \cdot x^3$, we immediately see that $x^2 \cdot x^3 = (x \cdot x) \cdot (x \cdot x \cdot x)$. And if we count up all the factors of x we see that $x^2 \cdot x^3 = x^5$. Therefore, so long as the bases are the same, we have the general rule: If m and n are positive integers and a is a real number, then

$$a^m \cdot a^n = a^{m+n}.$$

In our next example, we raise an exponential expression to a power. The expression $(x^2)^3$ means use x^2 as a factor three times. So $(x^2)^3 = x^2 \cdot x^2 \cdot x^2$. Applying our previous rule we have, $x^{2+2+2} = x^6$. Hence, $(x^2)^3 = x^6$. In general we write, If m and n are positive integers and a is a real number, then

$$(a^m)^n = a^{m \cdot n}$$

If we raise a product to a power, we raise every factor of the product to that power. In general we write, If a and b are real numbers and m is an integers, then

$$(ab)^m = a^m b^m$$

2.1 Example

Evaluate:

$$(3x^2y)^3 = 3^3(x^2)^3y^3 = 27x^6y^3$$

We may also raise a quotient to a power. Whenever we do this we raise the numerator and denominator separately and distinctly to that power.

2.2 Example

Evaluate:

$$(\frac{xy}{z})^3 = \frac{x^3y^3}{z^3}$$

In general we write, If m is an integer and a and b are real numbers, then

$$(\frac{a}{b})^m = \frac{a^m}{b^m}(b \neq 0)$$

3 Facts

1. When multiplying exponential expressions that have the same base, we may add the powers. $a^m \cdot a^m = a^{m+n}$
2. When raising a product to a power raise every factor of that product to that power $(ab)^m = a^m b^m$.

3. When raising a quotient to a power, raise the numerator and denominator separately and distinctly to that power. $(\frac{a}{b})^m = \frac{a^m}{b^m}$

4 Exercises

1. Simplify the expression using the power rule.

$$(-4x^2y)^3 =$$

2. Simplify the expression using the power rule.

$$(\frac{-2x^2y}{5z})^2 =$$

3. Simplify.

$$-2xy(x^2y)^3 - 5x^5(xy^2)^2 =$$

4. Simplify

$$3x^3(3x^2y)^2 - 4x(x^3y)^2 =$$

5 Solutions

1. Simplify the expression using the power rule.

$$(-4x^2y)^3 = (-4)^3x^5y^3 = -64x^6y^3$$

2. Simplify the expression using the power rule.

$$(\frac{-2x^2y}{5z})^2 = \frac{4x^4y^2}{25z^2}$$

3. Simplify.

$$-2xy(x^2y)^3 - 5x^5(xy^2)^2 = -2xyx^6y^3 - 5x^5x^2y^4$$

=

$$-2x^7y^4 - 5x^7y^4$$

4. Simplify

$$3x^3(3x^2y)^2 - 4x(x^3y)^2 = 3x^39x^4y^2 - 4xx^6y^2$$

=

$$27x^7y^2 - 4x^7y^2$$