

1 Polynomials 5.2

By the end of this section, you should be able to solve the following problems.

1. Identify the following as a monomial, binomial, trinomial, or other type of polynomial. What is the degree of the expression?

$$(1 - 2x) + 4x^2 - x^3$$

2. Simplify the given expression.

$$(5x^2 - 3y^2) + (3x^2 - 2xy + 6y^2)$$

3. Perform the indicated operation.

$$\textit{Subtract } 3m^3 + 5m + 5 \textit{ from } 4m^3 - 5m - 5$$

4. Perform the indicated subtraction.

$$(-9t^4 + 7t^2 - 1) - (-9t^4 + 8t^2 + 10)$$

2 Concepts

When we look at an expression like $5x^4 + 2x^3 - 7x + 4$, what we see are variables separated by plus and minus signs. Each one of those variable parts is called

a term. Within each term the number part is called the coefficient and the exponent is called the degree of the term. For instance, $5x^4$ has a coefficient of 5 and a degree of 4. Taken by itself, a single term is called a monomial. Two terms added together are called a binomial. A Three term expression is called a trinomial. More than three terms is simply called a polynomial. In any polynomial, the term of highest degree determines the degree of the polynomial. So we would say that $3x^4 - 2x^3 + x^2 - 7$ is a polynomial of degree 4. Note that it is customary to list the terms in a polynomial in order of degree from highest to lowest from left to right.

2.1 Example

State the degree of the polynomial, and arrange the terms in order of degree from left to right. List the coefficients of each term, including the constant term, in roster form.

$$2x^2 + 11 - 5x^3 - x$$

Answer. The degree of the polynomial is 3; $-5x^3 + 2x^2 - x + 11$; $\{-5, 2, -1, 11\}$.

3 Concepts

When we add two polynomials together, we add them term by term and like terms to like terms. Similarly we subtract polynomials by subtracting them term by term only subtracting like terms. In the next example, we add two polynomials.

3.1 Example

Add:

$$(4x^3 + 2x^2 + x - 10) + (2x^2 - 7x + 4) = 4x^3 + 4x^2 - 6x - 6$$

In the next example, we subtract two polynomials.

3.2 Example

Subtract:

$$8x^2 - 4x + 7 \text{ from } 9x^3 - 11x^2 + 10x - 3$$

We put the expression after the word *from* first.

$$(9x^3 - 11x^2 + 10x - 3) - (8x^2 - 4x + 7)$$

Next, we change the subtraction sign to addition and change all the signs in the expression to the right of the subtraction sign to their opposites. Then

we add like terms.

$$(9x^3 - 11x^2 + 10x - 3) + (-8x^2 + 4x - 7)$$

=

$$9x^3 - 19x^2 + 14x - 10$$

4 Facts

1. The coefficient of a term is the large number next to the variable.
2. The degree of a polynomial is the degree of term that has the highest power in the polynomial.
3. When adding or subtracting polynomials, we add or subtract like terms term by term.
4. If a problem is written: Subtract a from b , we rewrite it to say $b - a$.
5. When we subtract in algebra, we add the opposite.

5 Exercises

1. Identify which is a monomial, binomial, trinomial or other type of polynomial. What is the degree of the polynomial?

$$(1 - 2x) + 4x^2 - x^3$$

2. Simplify the given expression.

$$(5x^2 - 3y^2) + (3x^2 - 2xy + 6y^2)$$

3. Subtract

$$3m^3 + 5m + 5 \text{ from } 4m^3 - 5m - 5$$

4. Perform the indicated subtraction.

$$(-9t^4 + 7t^2 - 1) - (-9t^4 + 8t^2 + 10)$$

6 Solutions

1. Identify which is a monomial, binomial, trinomial or other type of polynomial. What is the degree of the polynomial?

$$(1 - 2x) + 4x^2 - x^3$$

This is a polynomial of degree 3.

2. Simplify the given expression.

$$(5x^2 - 3y^2) + (3x^2 - 2xy + 6y^2)$$

=

$$8x^2 - 2xy + 3y^2$$

3. Subtract

$$3m^3 + 5m + 5 \text{ from } 4m^3 - 5m - 5$$

$$4m^3 - 5m - 5 - (3m^3 + 5m + 5)$$

$$4m^3 - 5m - 5 + (-3m^3 - 5m - 5)$$

=

$$m^3 - 10m - 10$$

4. Perform the indicated subtraction.

$$(-9t^4 + 7t^2 - 1) - (-9t^4 + 8t^2 + 10)$$

=

$$(-9t^4 + 7t^2 - 1) + (9t^4 - 8t^2 - 10)$$

=

$$-t^2 - 11$$