

1 Multiplication of Polynomials 5.3

By the end of this section, you should be able to solve the following problems.

1. Use the distributive law to find the product and simplify the result.

$$-4z^2(5 + z)$$

2. Use the distributive law to find the product and simplify the result.

$$(6x + 2)(5x^2 - 7x)$$

3. Find the product and simplify.

$$(a^2 - 2a + 7)(a - 2)$$

4. Use the FOIL method to find the product.

$$(3x - 5)(2x + 5)$$

2 Concepts

In the last section, we learned how to add polynomials. Now we must learn how to multiply them. Multiplication of polynomials is nothing more than an application of the distributive law, and the distributive law dictates that

every term in one factor must be multiplied by every term in the other factor. The same idea applies in arithmetic. When we multiply 14 by 23. We could write this product in the following way:

$$(10 + 4)(20 + 3)$$

We then proceed to multiply 4 by 20 and 3 and 10 by 20 and 3. So we get:

$$(10 + 4)(20 + 3) = 4 \cdot 20 + 4 \cdot 3 + 10 \cdot 20 + 10 \cdot 3 = 80 + 12 + 200 + 30 = 322$$

No one would argue with these facts. The exact same idea is true for algebraic terms. In the following example, we multiply each term by each term.

2.1 Example

Use the distributive property to find the product:

$$(x + 2)(x + 3) = 2 \cdot x + 2 \cdot 3 + x \cdot x + x \cdot 3 = 2x + 6 + x^2 + 3x.$$

The next step is to collect like terms and put the powers of x in descending order from left to right. The result is

$$x^2 + 5x + 6$$

Usually a product like this is taught by using the acronym FOIL. This stands for First, Outer, Inner, Last. In the product,

$$(x + 2)(x + 3)$$

we multiply the first term of the first binomial by the first term of the second binomial to get x^2 then we multiply the outer terms of binomials together to get $3x$. The inner terms give $2x$, and the last terms of the binomials give 6. So

$$(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$$

2.2 Example

Similarly, we can multiply any number of terms together. In this example, we multiply two terms by three terms. Again we proceed in an each-by-each fashion.

$$(x + 2)(x^2 + 5x - 3) = x^3 + 5x^2 - 3x + 2x^2 + 10x - 6 = x^3 + 7x^2 + 7x - 6$$

It is very important to remember that the distributive law is unforgiving. Every term must be multiplied by every other term, and if one product is left out, the whole answer will be wrong.

3 Facts

1. Whenever we apply the distributive law, we must multiply every term by every other term.
2. When multiplying two binomials together we use the acronym FOIL which stands for First, Outer, Inner, Last which are the terms of the binomials. For example, $(x-3)(x+4) = x^2 + 4x - 3x - 12 = x^2 + x - 12$.
3. Whenever we multiply terms together, we must remember to add like terms if there are any.

4 Exercises

1. Use the distributive law to find the product and simplify the result.

$$-4z^2(5 + z)$$

2. Use the distributive law to find the product and simplify the result.

$$(6x + 2)(5x^2 - 7x)$$

3. Find the product and simplify.

$$(a^2 - 2a + 7)(a - 2)$$

4. Use the FOIL method to find the product.

$$(3x - 5)(2x + 5)$$

5 Solutions

1. Use the distributive law to find the product and simplify the result.

$$\begin{aligned} & -4z^2(5+z) \\ = & \\ & -20z^2 - 4z^3 \end{aligned}$$

2. Use the distributive to find the product and simplify the result.

$$\begin{aligned} & (6x+2)(5x^2-7x) \\ = & \\ & 30x^3 - 42x^2 + 10x^2 - 14x \\ & 30x^3 - 32x^2 - 14x \end{aligned}$$

3. Find the product and simplify.

$$\begin{aligned} & (a^2 - 2a + 7)(a - 2) = a^3 - 2a^2 + 7a - 2a^2 + 4a - 14 \\ = & \\ & a^3 - 4a^2 + 11a - 14 \end{aligned}$$

4. Use the FOIL method to find the product.

$$(3x-5)(2x+5) = 6x^2 + 15x - 10x - 25$$

=

$$6x^2 + 5x - 25$$