

1 Special Products 5.4

1. Write the following in expanded form.

$$(x + 4)^2$$

2. Write the following in expanded form.

$$(3a - 4b)^2$$

3. Use the sum and difference rule to find the product of the binomials.

$$(3x + 2)(3x - 2)$$

4. Use the sum and difference rule to find the product of the binomials.

$$(4 + r^2)(4 - r^2)$$

2 Concepts

There are certain products that occur so frequently in algebra that we set aside rules for them. We also want these patterns to be recognized on sight. Of course, if ever we forget the rule, it is easy to find the product, simply multiply the terms each-by-each.

Square of a Binomial

For all real numbers a and b ,

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

$$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$$

Product of a Binomial Sum and Difference

For all real numbers a and b ,

$$(a + b)(a - b) = a^2 - b^2$$

We now give two examples of the square of a binomial and the product of a sum and difference.

2.1 Examples

1. Use the square of a binomial rule to expand the following expressions.

(a)

$$(2x + 1)^2 = (2x + 1)(2x + 1) = 4x^2 + 4x + 1$$

(b)

$$(x - 3)^2 = (x - 3)(x - 3) = x^2 - 6x + 9$$

2. Use the binomial sum and difference rule to find the product.

(a)

$$(2x + 4)(2x - 4) = 4x^2 - 8x + 8x - 16 = 4x^2 - 16$$

Notice that whenever we apply the sum and difference rule, the middle term drops out.

(b)

$$(a + 6b)(a - 6b) = a^2 - 12ab + 12ab - 36b^2 = a^2 - 36b^2$$

3 Facts

Certain product patterns repeat over and over in mathematics so they must be memorized. The two discussed in this section are the *The Square of a Binomial* and *The Product of a Binomial Sum and Difference*.

Square of a Binomial

For all real numbers a and b ,

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

$$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$$

Product of a Binomial Sum and Difference

For all real numbers a and b ,

$$(a + b)(a - b) = a^2 - b^2$$

Also, remember that whenever we find the product of the sum and difference of two binomials, the middle term drops out.

4 Exercises

1. Write the following in expanded form.

$$(x + 4)^2$$

2. Write the following in expanded form.

$$(3a - 4b)^2$$

3. Use the sum and difference rule to find the product of the binomials.

$$(3x + 2)(3x - 2)$$

4. Use the sum and difference rule to find the product of the binomials.

$$(4 + r^2)(4 - r^2)$$

5 Solutions

1. Write the following in expanded form.

$$(x + 4)^2 = (x + 4)(x + 4) = x^2 + 8x + 16$$

2. Write the following in expanded form.

$$(3a - 4b)^2 = (3a - 4b)(3a - 4b) = 9a^2 - 24ab + 16b^2$$

3. Use the sum and difference rule to find the product of the binomials.

$$(3x + 2)(3x - 2) = 9x^2 - 4$$

4. Use the sum and difference rule to find the product of the binomials.

$$(4 + r^2)(4 - r^2) = 16 - r^4$$