1 Integer Exponents 5.5

By the end of this section, you should be able to solve the following problems.

1. Write the expression with positive exponents. Assume all variables are non-zero.

$$(-2x^2)(x^{-5})^2$$

2. Write the expression with positive exponents. Assume all variables are non-zero.

$$\frac{x^6(x^4)^{-2}}{(x^{-2})^2}$$

3. Write the expression with positive exponents. Assume all variables are non-zero.

$$\frac{2y^{-2}z^4}{(3y^2z^{-2})^2}$$

4. Simplify the numerical expression.

$$5 \cdot 4^{-2} - 3 \cdot 2^{-3}$$

2 Concepts

Definition

For all x $x(\neq 0)$

$$x^{-1} = \frac{1}{x}$$

Whenever we simplify an expression with negative exponents, the goal is always to eliminate the negative signs on the exponents. By definition, if a number is raised to a negative power and it is in a numerator, we eliminate the negative sign on the exponent by putting the whole factor in the denominator. Similarly, if a number is raised to a negative power and it is in a denominator, we eliminate the negative sign on the exponent by putting the whole factor in the numerator.

2.1 Example

Eliminate negative exponents and simplify.

$$(3x^2)^{-2}(x^2y)$$

Raise each factor to the -2 power

$$(3^{-2}x^{-4})(x^2y)$$

Eliminating negative exponents we have.

$$\frac{x^2y}{3^2x^4}$$

Reducing we have.

$$\frac{y}{9x^2}$$

Our next example has negative powers in the numerator and denominator. We eliminate negative exponents by putting the factor with the negative exponent in the numerator, if it is in the denominator, and in the denominator if it is in the numerator.

2.2 Example

Eliminate negative exponents and reduce.

$$\frac{2^{-2}x^{-2}y^{-3}}{2^{-4}x^{-5}y^{-10}}$$

We begin by swapping the expressions with negative exponents in the numerator with those in the denominator. This gives:

$$\frac{2^4x^5y^{10}}{2^2x^2y^3}$$

Now we reduce at will.

$$2^2x^3y^7$$

In our last example, we eliminate negative exponents in an expression that contains only numbers.

2.3 Example

Eliminate negative exponents.

$$5 \cdot 2^{-2} - 2^{-3} \cdot 3^2$$

$$\frac{5}{2^2} - \frac{3^2}{2^3}$$

$$\frac{5}{4} - \frac{9}{8}$$

$$(\frac{2}{2})(\frac{5}{4}) - \frac{9}{8}$$

$$\frac{10}{8} - \frac{9}{8}$$

$$\frac{1}{8}$$

3 Facts

1. We eliminate negative exponents by putting factors raised to negative powers in the denominator if they are in the numerator, and in the numerator if they are in the denominator.

2. An expression raised to a negative power is the reciprocal of that expression.

3. Always eliminate negative exponents before reducing rational expressions.

4. *Definition*

   For all x $(x \neq 0)$

$$x^{-1} = \frac{1}{x}$$

4 Exercises

1. Write the expression with positive exponents. Assume all variables are non-zero.

$$(-2x^2)(x^{-5})^2$$

2. Write the expression with positive exponents. Assume all variables are non-zero.

$$\frac{x^5(x^4)^{-2}}{(x^{-2})^2}$$

3. Write the expression with positive exponents. Assume all variables are non-zero.

$$\frac{2y^{-2}z^4}{(3y^2z^{-2})^2}$$

4. Simplify the numerical expression.

$$5 \cdot 4^{-2} - 3 \cdot 2^{-3}$$

5 Solutions

1. Write the expression with positive exponents. Assume all variables are non-zero.

$$(-2x^2)(x^{-5})^2$$

$$\frac{-2x^2}{x^{10}}$$

$$\frac{-2}{x^8}$$

2. Write the expression with positive exponents. Assume all variables are non-zero.

$$\frac{x^5(x^4)^{-2}}{(x^{-2})^2}$$

$$\frac{x^{-3}}{x^{-4}}$$

$$\frac{x^4}{x^3}$$

$$x$$

3. Write the expression with positive exponents. Assume all variables are non-zero.

$$\frac{2y^{-2}z^4}{(3y^2z^{-2})^2}$$

$$\frac{2y^{-2}z^4}{3^2y^4z^{-4}}$$

$$\frac{2z^4z^4}{3^2y^4y^2}$$

$$\frac{2z^8}{9y^6}$$

4. Simplify the numerical expression.

$$5 \cdot 4^{-2} - 3 \cdot 2^{-3}$$

$$\frac{5}{16} - (\frac{3}{8})(\frac{2}{2})$$

$$\frac{5}{16} - \frac{6}{16}$$

$$\frac{-1}{16}$$