1 Quotient of Two Polynomials 5.6

By the end of this section, you should be able to solve the following problems.

1. Find the quotient.
$$\frac{22a^2b + 11ab - 44ab^2}{11ab}$$

2. Find the quotient.
$$\frac{9p^2q + 6pq - 3pq^2}{pq}$$

3. Use long division to find the quotient and remainder.
$$(x^2 - 14x + 49) \div (x - 7)$$

4. Use long division to find the quotient and remainder.
$$(y^2 - y - 6) \div (y - 3)$$

2 Concepts

In this treatment, we will divide a polynomial by a monomial by using a technique called distributing the denominator. Many students believe that they can safely omit this step, but to do so is to risk making the mistake of cancellation across addition. Whenever students do this the error is always the same, they cancel the denominator more times than is allowed for each individual term in the numerator. We will not brook this fallacy. Therefore, whenever we divide a polynomial by a monomial we must distribute the denominator.

2.1 Example

Divide:

$$\frac{x^4 + 5x^3 + 10x^2 + x - 4}{2x}$$

First we distribute the denominator, and then we do all our cancellations within terms.

$$\frac{x^4}{2x} + \frac{5x^3}{2x} + \frac{10x^2}{2x} + \frac{x}{2x} - \frac{4}{2x}$$

+

$$\frac{x^3}{2} + \frac{5x^2}{2} + 5x + \frac{1}{2} - \frac{2}{x}$$

3 Concepts

In our next example, we must divide a polynomial by a binomial. Here we do long division until there is no more variable left. The algorithm is the same as it was in grade school. First we divide, then we multiply, then we subtract, then we bring down. We continue this process until the variable has been divided out completely. In our next example, there is no remainder.

3.1 Example

Divide:

$$(x^2 + 5x + 6) \div (x + 2)$$

First we rewrite the problem in the following way.

$$\begin{array}{r} x \\ x+2 \overline{) x^2 + 5x + 6} \\ \underline{-(x^2 + 2x)} \\ 3x + 6 \end{array}$$

Then we divide x^2 by x, and we place the quotient over $5x$ in the dividend.

Then we multiply $x + 2$ by x and subtract the product from x^2 and $5x$.

Notice that we add the opposite of $x^2 + 2x$ to $x^2 + 5x$ and bring down the 6. The next step is to divide $3x$ by x. We illustrate the rest of the problem below.

$$\begin{array}{r} x + 3 \\ x+2 \overline{) x^2 + 5x + 6} \\ \underline{-(x^2 + 2x)} \\ 3x + 6 \\ \underline{-(3x + 6)} \\ 0 \end{array}$$

Our next example contains a remainder.

3.2 Example

$$
\begin{array}{r}
x + 4 \qquad + \frac{2}{x+1} \\
x + 1 \overline{) \, x^2 + 5x + 6} \\
-(x^2 + x) \\
\hline
4x + 6 \\
-(4x + 6) \\
\hline
2
\end{array}
$$

4 Facts

1. Whenever we divide a polynomial by a monomial, we must distribute the denominator.

2. Whenever we divide a polynomial by a binomial we perform a four step precess. (1) Divide by the divisor (2) Multiply the quotient by the entire binomial (3) Subtract that product from dividend. (4) Bring down the next term.

3. If there is a remainder, simply write that as a ratio added in at the end of the quotient.

5 Exercises

1. Find the quotient.

$$\frac{22a^2b + 11ab - 44ab^2}{11ab}$$

2. Find the quotient.

$$\frac{9p^2q + 6pq - 3pq^2}{pq}$$

3. Use long division to find the quotient and remainder.

$$(x^2 - 14x + 49) \div (x - 7)$$

4. Use long division to find the quotient and remainder.

$$(y^2 - y - 6) \div (y - 3)$$

6 Solutions

1. Find the quotient.

$$\frac{22a^2b + 11ab - 44ab^2}{11ab}$$

=

$$\frac{22a^2b}{11ab} + \frac{11ab}{11ab} - \frac{44ab^2}{11ab}$$

=

$$2a + 1 - 4b$$

2. Find the quotient.

$$\frac{9p^2q + 6pq - 3pq^2}{pq}$$

=

$$\frac{9p^2q}{pq} + \frac{6pq}{pq} - \frac{3pq}{pq}$$

=

$$9p + 6 - 3$$

3. Use long division to find the quotient and remainder.

$$(x^2 - 14x + 49) \div (x - 7)$$

$$
\begin{array}{r}
x-7 \\
x-7 \overline{\smash{\big)}\, x^2-14x+49} \\
\underline{-(x^2-7x)} \\
-7x+49 \\
-(\underline{-7x+49}) \\
0
\end{array}
$$

4. Use long division to find the quotient and remainder.

$$(y^2-y-6) \div (y-3)$$

$$
\begin{array}{r}
y+2 \\
y-3 \overline{\smash{\big)}\, y^2-y-6} \\
\underline{-(y^2-3y)} \\
2y-6 \\
\underline{\text{-}(2y-6)} \\
0
\end{array}
$$