1 Factors, Greatest Common Factor 6.1

By the end of this section, you should be able to solve the following problems.

1. Find the greatest common factor of:

$$15m, 12n^2, 30p$$

2. Factor out the greatest common factor and write the expression in factored form. Use the distributive property to verify your answer.

$$9x^3 - 12x^5 + 24x^6$$

3. Factor out the "minus" sign and any other common factor.

$$-3y + 3$$

4. Factor by grouping.

$$x^2y - x^2 - 3y + 3$$

2 Concepts

The greatest common factor among a set of numbers is built by taking the product of the prime numbers common to all the numbers. To find the greatest common factor, we first write the numbers as products of primes.

2.1 Example

Find the greatest common factor (GCF) of 16, 24, and 30

First we break up the numbers into products of their prime factors.

$$16 = 2 \cdot 2 \cdot 2 \cdot 2$$

$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$

$$30 = 2 \cdot 3 \cdot 5$$

From the above factorizations, we see that the only prime that is common to all three numbers is 2, and the maximum number of times 2 occurs is all three lists simultaneously is once, therefore the GCF is 2. In our next example, we find the GCF among a set of algebraic terms.

2.2 Example

Find the greatest common factor of

$$12x^4, 30x^3, 24x^2$$

.

Again, we list all the prime factors of the terms.

$$12x^4 = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x$$

$$30x^3 = 2 \cdot 3 \cdot 5 \cdot x \cdot x \cdot x$$

$$24x^2 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x$$

In the lists above, we look for the maximum number of common factors that occur simultaneously. Those are,

$$2 \cdot 3 \cdot x \cdot x$$

.

So the greatest common factor is

$$6x^2$$

In our next example, we factor out the greatest common factor and write the expression in factored form.

2.3 Example

Factor the expression.

$$8x^4y^3 - 6x^3y^4 + 10x^2y^5$$

First we look for the largest number that divides all three coefficients. That would be 2. Then we look for the highest power of x that divides x^2, x^3, x^4, and that would be x^2. Finally, the highest power of y that divides all three terms is y^3. Writing the expression in factored form we have:

$$2x^2y^3(4x^2 - 3xy + 5y^3)$$

An important trick to be able to handle in algebra is to able to factor out -1 when it may not look like -1 is common factor. But this can always be done.

2.4 Example

Factor -1 out of the expression.

$$x - 1 = -1 \cdot (1 - x)$$

We can certainly check this by the distributive law.

$$-1 \cdot (1 - x) = (-1) \cdot 1 + (-1) \cdot -x = -1 + x = x - 1$$

.

In our next example, we use a technique called factoring by grouping. Here we group specific terms together and then find the GCF of just those grouped terms.

2.5 Example

Factor the expression by grouping.

$$2x^2 + 2x - 3x - 3$$

In the expression above we group the first two terms and the last two terms using parentheses.

$$(2x^2 + 2x) + (-3x - 3)$$

Notice that we inserted a plus sign between the grouped pairs. We never separate a negative signs from their coefficients. In our next step, we factor the respective pairs.

$$2x(x + 1) - 3(x + 1)$$

Notice the common binomial, (x+1), we will now factor that out.

$$(x + 1)(2x - 3)$$

Now the expression is completely factored.

3 Facts

1. The Greatest Common Factor (GCF) is the largest number or expression that evenly divides all the numbers or terms.

2. We find the GCF by listing the maximum number of primes that are common to all the terms or numbers and then and multiply them together.

3. We can always factor -1 out of any binomial. Look:

$$a - b = -1 \cdot (b - a)$$

4. To factor by grouping, group the first two terms of an expression and the last two terms of the expression together and insert a plus sign in between. Never separate a negative sign from its coefficient.

4 Exercises

1. Find the greatest common factor.

$$15m, 12n^2, 30p$$

2. Factor out the greatest common factor and write the expression in factored form. Use the distributive property to verify your answer.

$$9x^3 - 12x^5 + 24x^5$$

3. Factor out the "minus" sign and any other common factor.

$$-3y + 3 = -3(y - 1)$$

4. Factor by grouping.

$$x^2y - x^2 - 3y + 3$$

5 Solutions

1. Find the greatest common factor.

$$15m, 12n^2, 30p$$

The greatest common factor is 3.

2. Factor out the greatest common factor and write the expression in factored form. Use the distributive property to verify your answer.

$$9x^3 - 12x^5 + 24x^5$$

=

$$3x^3(3 - 4x^2 + 8^3)$$

3. Factor out the "minus" sign and any other common factor.

$$-3y + 3 = -3(y - 1)$$

4. Factor by grouping.

$$x^2y - x^2 - 3y + 3$$

=

$$(x^2y - x^2) + (-3y + 3)$$

$=$

$$x^2(y-1)-3(y-1)$$

$=$

$$(y-1)(x^2-3)$$