1 Factoring Trinomials 6.2

By the end of this section, you should be able to solve the following problems.

1. Factor the trinomial.

$$y^2 + y - 56$$

2. Factor the trinomial.

$$x^2 - 8x - 65$$

3. Factor the trinomial.

$$2x^2 + 9x + 10$$

4. Factor the trinomial.

$$8x^2 - 7x - 3$$

2 Concepts

A quadratic expression written is standard from is

$$ax^2 + bx + c.$$

Our first job will be to learn how to factor these expressions when the coefficient of the x^2 term is 1. There are recurring patterns in these expressions. Let is begin with some perfect square trinomials.

2.1 Example

Notice the pattern of signs in the following perfect square trinomials.

$$x^2 + 10x + 25$$

$$x^2 - 10x + 25$$

In both the expressions above, the sign before the constant term is positive. Whenever this sign is positive, the signs of the binomial factors will be the same. Now look at the sign of the middle term in both. In the first expression it is positive and in the second it is negative. This tells us that the signs of the binomial factors for the first expression will both be positive, and the signs of the binomial factors for the second expression will both be negative.

$$x^2 + 10x + 25x = (x + 5)(x + 5)$$

$$x^2 - 10x + 25x = (x - 5)(x - 5)$$

In our next example, the sign before the constant term will be negative. This tells us that one of the signs between binomial factors will be positive and the other will be negative.

2.2 Example

$$x^2 - x - 6 = (x + 2)(x - 3)$$

Notice here that the sign of the middle term is negative, therefore, the larger of the inner and outer products must be negative. Now we turn to factoring trinomial where the coefficient of the x^2 term is not 1.

2.3 Example

Factor:

$$3x^2 + 2x - 1$$

We start by finding the product of the lead coefficient and the constant term. $3 \cdot (-1) = -3$. Now we look for factors of -3 whose difference is +2 (the coefficient of the middle term). What will do this for us is 3+(-1). Our next job is to rewrite the middle term in as a sum with coefficients 3 and (-1). We do this below.

$$3x^2 + 3x - x - 1$$

Now we group terms to exploit factoring by grouping.

$$(3x^2 + 3x) + (-x - 1)$$

$=$

$$3x(x+1) - 1 \cdot (x+1)$$

$=$

Now we can factor out a common $x + 1$.

$$3x(x+1) - 1 \cdot (x+1) = (x+1)(3x-1)$$

3 Facts

1. The pattern $x^2 + cx + d$ factors to be binomials with the following sign pattern $(x + a)(x + b)$.

2. The pattern $x^2 - cx - d$ factors to be binomials with the following sign pattern $(x + a)(x - b)$. $(bx > ax.)$

3. The pattern $x^2 + cd - d$ factors to be binomials with the following sign pattern$(x + a)(x - b)$. $(ax > bx.)$

4. When factoring a trinomial where the lead coefficient is not 1, find the product of the lead coefficient and the constant term and decide on factors of that product that will add so the middle term can be expressed as a sum of those factors. Then, follow the method of factoring

by grouping.

4 Exercises

1. Factor the trinomial.

$$y^2 + y - 56$$

2. Factor the trinomial.

$$x^2 - 8x - 65$$

3. Factor the trinomial completely over the integers.

$$2x^2 + 9x + 10$$

4. Factor the trinomial completely over the integers.

$$6x^2 - 7x - 3$$

5 Solutions

1. Factor the trinomial.

$$y^2 + y - 56$$

=

$$(y + 8)(y - 7)$$

2. Factor the trinomial.

$$x^2 - 8x - 65$$

=

$$(x + 5)(x - 13)$$

3. Factor the trinomial completely over the integers.

$$2x^2 + 9x + 10$$

=

$$2x^2 + 4x + 5x + 10$$

=

$$2x(x + 2) + 5(x + 2)$$

$$(x + 2)(2x + 5)$$

4. Factor the trinomial completely over the integers.

$$6x^2 - 7x - 3$$

$=$

$$6x^2 - 9x + 2x - 3$$

$=$

$$3x(2x - 3) + 1 \cdot (2x - 3)$$

$=$

$$(2x - 3)(3x + 1)$$