1 Solving Quadratic Equations by Factoring

6.4

By the end of this section, you should be able to solve the following problems.

1. Solve the equation.

$$2x^2 - 18 = 0$$

2. Solve the equation.

$$x^2 + 9x + 20 = 0$$

3. Solve the equation.

$$10x^2 - 7x - 12 = 0$$

4. Solve the equation.

$$x^2 = \frac{1}{5}(32x - 2)$$

2 Concepts

All the previous sections on factoring culminate in this one. Now we apply what we know about factoring expressions to solving equations of the form $ax^2+bx+c = 0$. We will give two examples below. In the first, the coefficient

of the quadratic term will be one and in the second there will be a common numerical factor that we can factor out.

3 Example

Find the roots (solutions) to the quadratic equation.

$$6x^2 + 5x - 4 = 0$$

First we multiply 6 by (-4) and look for factors of -24 that add to +5. They are +8 and -3. Now we rewrite the middle term and factor by grouping.

$$6x^2 + 8x - 3x - 4 = 0$$

$$(6x^2 + 8x) + (-3x - 4) = 0$$

$$2x(3x + 4) - 1 \cdot (3x + 4) = 0$$

$$(3x + 4)(2x - 1) = 0$$

Now we use the zero factor principle to find the roots of the equation. The zero factor principle states that in the equation.

$$A \cdot B = 0$$

If either A or B is zero then it is a solution as to the equation. In our example, we find the roots by setting each factor of $(3x+4)(2x-1)=0$ equal to zero, separately and distinctly, and solving for x.

$$3x+4=0$$

$$-4 \quad -4$$

$$\overline{3x=-4}$$

$$\frac{3x}{3}=\frac{-4}{3}$$

Therefore, $\frac{-4}{3}$ is a solution or root of the equation.

To find the other root of the equation we set $2x-1=0$

$$2x-1=0$$

$$+1 \quad +1$$

$$\overline{2x=1}$$

$$\frac{2x}{2}=\frac{1}{2}$$

$$x=\frac{1}{2}$$

Therefore, $\frac{1}{2}$ is also a solution or root of the equation.

In the next example, we factor out a common numerical factor before factoring and finding the roots of the quadratic.

3.1 Example

Find the roots of the equation.

$$10x^2 + 35x - 20 = 0$$

The first thing we notice is that there is a factor of 5 common to all the terms. So we factor it out and divide both sides by 5 .

$$5(2x^2 + 7x - 4) = 0$$

$$\frac{5(2x^2 + 7x - 4)}{5} = \frac{0}{5}$$

$$2x^2 + 7x - 4 = 0$$

Now we factor by grouping. First we rewrite the middle term as a sum whose addends are factors of -8.

$$2x^2 + 8x - x - 4 = 0$$

Now we group factors and introduce a plus sign to maintain the additive structure.

$$(2x^2 + 8x) + (-x - 4) = 0$$

Now we factor what is common to the first two terms and the last two terms.

$$2x(x + 4) - 1 \cdot (x + 4) = 0$$

Factoring we have.

$$(x+4)(2x-1)=0$$

Setting each factor equal to zero, we identify the roots of the equations

$$x+4=0$$

$$-4 \quad -4$$

$$\overline{x=-4}$$

and

$$2x-1=0$$

$$-1 \quad -1$$

$$\overline{2x=-1}$$

$$\frac{2x}{2}=\frac{-1}{2}$$

$$x=\frac{-1}{2}$$

4 Facts

1. The zero factor property states that in the equation.

$$A \cdot B = 0$$

either $A=0$ *or* $B=0$

2. To find the roots of a quadratic equation, set each factor of the quadratic equal to zero and solve for the given variable.

3. When solving a homogeneous quadratic equation, there are usually two roots, except when the quadratic is a perfect square trinomial.

5 Exercises

1. Solve the equation

$$2x^2 - 18 = 0$$

2. Solve the equation.

$$x^2 + 9x + 20 = 0$$

3. Solve the equation.

$$10x^2 - 7x - 12 = 0$$

4. Solve the equation.

$$x^2 = \frac{1}{5}(32x - 12)$$

6 Solutions

1. Solve the equation

$$2x^2 - 18 = 0$$

$$2(x^2 - 9) = 0$$

$$\frac{2(x^2 - 9)}{2} = \frac{0}{2}$$

$$(x + 3)(x - 3) = 0$$

Therefore, $x = -3$ *or* $x = 3$.

2. Solve the equation.

$$x^2 + 9x + 20 = 0$$

$$(x + 4)(x + 5) = 0$$

Therefore, $x = -4$ or $x = -5$

3. Solve the equation.

$$10x^2 - 7x - 12 = 0$$

$$(10x^2 - 15x) + (8x - 12) = 0$$

$$5x(2x - 3) + 4(2x - 3) = 0$$

$$(2x - 3)(5x + 4) = 0$$

$$2x - 3 = 0$$

$$+3 \quad +3$$

$$\overline{2x = 3}$$

$$\frac{2x}{2} = \frac{3}{2}$$

$$x = \frac{3}{2}$$

$$5x + 4 = 0$$

$$-4 \quad -4$$

$$\overline{5x = -4}$$

$$\frac{5x}{5} = \frac{-4}{5}$$

$$x = \frac{-4}{5}$$

4. Solve the equation.

$$x^2 = \frac{1}{5}(32x - 12)$$

$$5x^2 = (5)\frac{1}{5}(32x - 12)$$

$$5x^2 = 32x - 12$$

$$-32x + 12 \quad -32x + 12$$

$$\overline{5x^2 - 32x + 12 = 0}$$

$$5x^2 - 30x - 2x + 12 = 0$$

$$(5x^2 - 30x) + (-2x + 12) = 0$$

$$5x(x - 6) - 2(x - 6) = 0$$

$$(x - 6)(5x - 2) = 0$$

Therefore, $x = 6$ or $x = \frac{2}{5}$