

1 Simplifying Rational Expressions 7.2

By the end of this section, you should be able to solve the following problems.

1. Write the rational expression in lowest terms.

$$\frac{36m^4v^5}{45m^3v^6}$$

2. Simplify the rational expression.

$$\frac{3x - 6}{x^2 - 4x + 4}$$

3. Simplify the rational expression.

$$\frac{x^2 + x - 6}{x^2 + 6x + 9}$$

4. Simplify the rational expression.

$$\frac{2t^3 + 19t^2 - 21t}{2t^2 + 21t}$$

2 Concepts

Any rational expression is completely simplified when there are no common factors in its numerator and denominator. For example, the fraction,

$\frac{18}{12}$, can be simplified because the numerator and denominator share a common factor of 6. In general, any ratio of the form $\frac{k \cdot a}{k \cdot b}$ where $(k, b \neq 0)$ can be reduced. The following examples will illustrate how to reduce rational expressions.

3 Example

1. Reduce the expression to lowest terms.

$$\begin{aligned} & \frac{18m^4n^5}{12m^2n^3} \\ & \frac{18m^4n^5 \div 6m^2n^3}{12m^2n^3 \div 6m^2n^3} \\ & = \\ & \frac{3m^2n^2}{2} \end{aligned}$$

2. Reduce to lowest terms.

$$\begin{aligned} & \frac{3x^2 - 2x - 1}{x^2 - 1} \\ & \frac{(3x^2 - 3x) + (x - 1)}{(x + 1)(x - 1)} \\ & \frac{3x(x - 1) + 1 \cdot (x - 1)}{(x + 1)(x - 1)} \end{aligned}$$

$$\frac{(x-1)(3x+1)}{(x+1)(x-1)}$$

$$\frac{3x+1}{x+1}$$

3. Reduce to lowest terms.

$$\frac{5x^2 - 9x - 2}{3x^2 - 10x + 8}$$

$$\frac{(5x^2 - 10x) + (x - 2)}{(3x^2 - 6x) + (-4x + 8)}$$

$$\frac{5x(x - 2) + 1 \cdot (x - 2)}{3x(x - 2) - 4(x - 2)}$$

$$=$$

$$\frac{(x - 2)(5x + 1)}{(x - 2)(3x - 4)}$$

$$=$$

$$\frac{5x + 1}{3x - 4}$$

4 Facts

1. Any rational expression is completely simplified when there are no factors in common between the numerator and denominator.
2. Always make sure the numerator and denominator are completely factored before you do any cancellation. You must always have pure mul-

multiplication in the numerator and denominator before reducing. Never cancel across addition.

5 Exercises

1. Write the rational expression in lowest terms.

$$\frac{36m^4v^5}{45m^3v^6}$$

2. Simplify the rational expression.

$$\frac{3x - 6}{x^2 - 4x + 4}$$

=

3. Simplify the rational expression.

$$\frac{x^2 + x - 6}{x^2 + 6x + 9}$$

=

4. Simplify the rational expression

$$\frac{2t^3 + 19t^2 - 21t}{2t^2 + 2 + 21t}$$

6 Solutions

1. Write the rational expression in lowest terms.

$$\frac{36m^4v^5}{45m^3v^6}$$

$$\frac{4m}{5v}$$

2. Simplify the rational expression.

$$\frac{3x - 6}{x^2 - 4x + 4}$$

=

$$\frac{3(x - 2)}{(x - 2)(x - 2)}$$

=

$$\frac{3}{x - 2}$$

3. Simplify the rational expression.

$$\frac{x^2 + x - 6}{x^2 + 6x + 9}$$

=

$$\frac{(x + 3)(x - 2)}{(x + 3)(x + 3)}$$

$$\frac{x - 2}{x + 3}$$

4. Simplify the rational expression

$$\frac{2t^3 + 19t^2 - 21t}{2t^2 + 2 + 21t}$$

$$\frac{t(2t^2 + 19t - 21)}{t(2t + 21)}$$

$$\frac{t((2t^2 - 2t) + (21t - 21))}{t(2t + 21)}$$

$$\frac{t(2t(t - 1) + 21(t - 1))}{t(2t + 21)}$$

$$\frac{t(t - 1)(2t + 21)}{t(2t + 21)}$$

=

$$t - 1$$