

# 1 Multiplying and Dividing Rational Expressions 7.3

By the end of this section, you should be able to solve the following problems.

1. Perform the indicated operation and simplify.

$$\frac{x^2 - 9}{x + 2} \cdot \frac{x^2 - 4}{x - 3}$$

2. Perform the indicated operation and simplify.

$$\frac{8z - 12}{12z + 7} \cdot \frac{42z + 21}{32z - 48}$$

3. Perform the indicated operation and simplify.

$$\frac{x^2 - 36}{x^2 - 25} \div \frac{x + 6}{x + 5}$$

4. Perform the indicated operation and simplify.

$$\frac{2x^2 - 7x - 4}{20 - x - x^2} \div \frac{2x^2 - 9x - 5}{x^2 - 5}$$

## 2 Concepts

When multiplying or dividing rational expressions in algebra, the idea is the same as when we multiply or divide fractions in arithmetic. Before we

multiply or divide, we reduce the expressions or fractions as much as possible.

Below are examples of multiplication with rational expressions.

## 2.1 Examples

$$\frac{18x^4y^3}{10x^5y^2} \cdot \frac{5xy}{3y^2} = 3$$

After some regrouping in the numerator and denominator, this is a little easier to see.

$$\frac{18 \cdot 5 \cdot x^4 \cdot x \cdot y^3 \cdot y}{10 \cdot 3 \cdot x^5 \cdot y^2 \cdot y^2}$$

Among the numbers, everything cancels except 3 in the numerator (30 goes into 90 three times). Then, 5 factors of  $x$  in the numerator cancel with 5 factors of  $x$  in the denominator, and 4 factors of  $y$  in the numerator cancel with 4 factors of  $y$  in the denominator.

In the next example, we must factor first before we multiply.

## 2.2 Example

Simplify the following expression.

$$\frac{2x + 2}{x^2 - 9} \cdot \frac{x^2 - x - 12}{x^2 - 1}$$

$$\frac{2(x+1)}{(x+3)(x-3)} \cdot \frac{(x+3)(x-4)}{(x+1)(x-1)}$$

Now that both ratios are in factored form, we may cancel at will. After cancelling,  $x+1$  and  $x+3$  in the numerator and denominator, we have:

$$\frac{2(x-4)}{(x-3)(x-1)}$$

### 3 Concepts

In our next example, we divide rational expressions. Just as with fractions, we invert the divisor and multiply. The reason for this is that any division that can be written as  $\frac{a}{b}$  can be rewritten  $a \cdot \frac{1}{b}$ . Therefore, if  $b$  is a fraction, we take its reciprocal and multiply. Below we invert the divisor before we factor and multiply.

#### 3.1 Example

Simplify the following expression.

$$\frac{2x-4}{3x^2+16x+5} \div \frac{x-2}{x^2+10x+25}$$

$$\frac{2x-4}{3x^2+16x+5} \cdot \frac{x^2+10x+25}{x-2}$$

$$\frac{2(x-2)}{3x^2+15x+x+5} \cdot \frac{(x+5)(x+5)}{x-2}$$

$$\frac{2(x-2)}{3x(x+5)+1 \cdot (x+5)} \cdot \frac{(x+5)(x+5)}{x-2}$$

$$\frac{2(x-2)}{(x+5)(3x+1)} \cdot \frac{(x+5)(x+5)}{x-2}$$

=

$$\frac{2(x+5)}{3x+1}$$

## 4 Facts

1. When we multiply or divide two rational expressions, we must always reduce the the expressions as much as possible before we multiply or divide.
2. Often, to reduce rational expressions before we divide or multiply, we must factor expressions in the numerator and denominator.
3. Whenever we divide rational expressions, we must take the reciprocal of the divisor and then multiply the two expressions together.

## 5 Exercises

1. Perform the indicated operation and simplify.

$$\frac{x^2 - 9}{x + 2} \cdot \frac{x^2 - 4}{x - 3}$$

2. Perform the indicated operation and simplify.

$$\frac{8z - 12}{14z + 7} \cdot \frac{42z + 21}{32z - 48}$$

3. Perform the indicated operation and simplify.

$$\frac{x^2 - 36}{x^2 - 25} \div \frac{x + 6}{x + 5}$$

4. Perform the indicated operation and simplify.

$$\frac{2x^2 - 7x - 4}{20 - x - x^2} \div \frac{2x^2 - 9x - 5}{x^2 - 5}$$

## 6 Solutions

1. Perform the indicated operation and simplify.

$$\frac{x^2 - 9}{x + 2} \cdot \frac{x^2 - 4}{x - 3}$$

=

$$\frac{(x + 3)(x - 3)}{x + 2} \cdot \frac{(x + 2)(x - 2)}{x - 3}$$

$$(x + 3) \cdot (x - 2)$$

=

2. Perform the indicated operation and simplify.

$$\frac{8z - 12}{14z + 7} \cdot \frac{42z + 21}{32z - 48}$$

=

$$\frac{(2z - 3)}{(2z + 1)} \cdot \frac{3(2z + 1)}{4(2z - 3)}$$
$$\frac{3}{4}$$

3. Perform the indicated operation and simplify.

$$\frac{x^2 - 36}{x^2 - 25} \div \frac{x + 6}{x + 5}$$

=

$$\frac{(x + 6)(x - 6)}{(x + 5)(x - 5)} \cdot \frac{x + 5}{x + 6}$$

=

$$\frac{x-6}{x-5}$$

4. Perform the indicated operation and simplify.

$$\frac{2x^2 - 7x - 4}{20 - x - x^2} \div \frac{2x^2 - 9x - 5}{x^2 - 5}$$

$$\frac{2x^2 - 8x + x - 4}{(5+x)(4-x)} \cdot \frac{x^2 - 5}{2x^2 + x - 10x - 5}$$

=

$$\frac{2x(x-4) + 1 \cdot (x-4)}{(5+x)(4-x)} \cdot \frac{(x+5)(x-5)}{x(2x+1) - 5(2x+1)}$$

=

$$\frac{(x-4)(2x+1)}{(5+x)(4-x)} \cdot \frac{(x+5)(x-5)}{(2x+1)(x-5)}$$

=

$$\frac{(x-4)(2x+1)}{-(x-4)(5+x)} \cdot \frac{(x+5)(x-5)}{(2x+1)(x-5)}$$

=

$$-1$$