1 Adding and Subtracting Rational Expressions 7.5

By the end of this section, you should be able to solve the following problems.

1. Add the rational expressions with different denominators.

$$\frac{2-t}{9t+6}+\frac{t-2}{6t+4}$$

2. Add the rational expressions with different denominators.

$$\frac{2a}{4a^2-9b^2}+\frac{1}{(2a-3b)}$$

3. Subtract the rational expressions.

$$\frac{6}{2a+2}-\frac{a+4}{4a-4}$$

4. Subtract the rational expressions.

$$\frac{3a}{9a^2-4b^2}-\frac{1}{3a+2b}$$

2 Concepts

When adding or subtracting rational expressions, just as with numerical fractions, you must get a common denominator. To get a common denominator, it is often necessary to factor denominators in order "see" the factors that will build a least common denominator. In our first example, we will add rational expressions and we will factor the denominators to see what factors compose the lowest common denominator.

2.1 Example

Add.

$$\frac{4x}{x^2+2x+1}+\frac{2}{x^2-2x-3}$$

First, we factor the denominators to see what factors we will need to build a least common denominator.

$$\frac{4x}{(x+1)(x+1)}+\frac{2}{(x-3)(x+1)}$$

The lowest common denominator must be divisible by two factors of $x+1$ and one factor of $x-3$, so it will be the product of $(x+1)(x+1)(x-3)$. Now, we must write each rational expression in terms of the lowest common denominator.

$$\frac{(x-3)4x}{(x-3)(x+1)(x+1)}+\frac{(x+1)2}{(x-3)(x+1)(x+1)}$$

=

$$\frac{4x^2 - 12x + 2x + 2}{(x-3)(x+1)(x+1)}$$

=

$$\frac{4x^2 - 10x + 2}{(x-3)(x+1)(x+1)}$$

In the next example, we subtract rational expressions. What most students fail to realize is when we subtract, and there is a numerator of more that one term in the subtrahend, we must distribute a negative one across all the terms in the numerator.

2.2 Example

subtract.

$$\frac{x^2+1}{x^2-x-6} - \frac{x+2}{x^2-2x-3}$$

Now we factor the denominators.

$$\frac{x^2+1}{(x-3)(x+2)} - \frac{x+2}{(x+1)(x-3)}$$

The least common denominator for this difference will be $(x-3)(x+2)(x+1)$.

$$\frac{(x+1)(x^2+1)}{(x+1)(x+2)(x-3)} + \frac{-1 \cdot (x+2)(x+2)}{(x+2)(x+1)(x-3)}$$

=

$$\frac{x^3 + x^2 + x + 1 - x^2 - 4x - 4}{(x+1)(x+2)(x-3)}$$

=

$$\frac{x^3 - 3x - 3}{(x+1)(x+2)(x-3)}$$

3 Facts

1. Whenever we add or subtract rational expression we must always get a common denominator.

2. If we are subtracting, and the numerator of the subtrahend has more than one term, the negative sign must be distributed across all the terms. The next step is to collect like terms.

3. The lowest common denominator LCD is usually found by factoring the denominators and "building" the LCD from the factors. The LCD will always be the product of the relatively prime factors in the denominators. Each distinct factor must occur in the LCD the maximum number of times it occurs in any denominator.

4 Exercises

1. Add the rational expression, with different denominators.

$$\frac{2-t}{9t+6} + \frac{t-2}{6t+4}$$

2. Add the rational expression, with different denominators.

$$\frac{2a}{4a^2-9b^2} + \frac{1}{2a-3b}$$

3. Subtract the rational expressions.

$$\frac{6}{2a+2} - \frac{a+4}{4a-4}$$

4. Subtract the rational expressions.

$$\frac{3a}{9a^2-4b^2} - \frac{1}{3a+2b}$$

5 Solutions

1. Add the rational expression, with different denominators.

$$\frac{2-t}{9t+6}+\frac{t-2}{6t+4}$$

$$\frac{-(t-2)}{3(3t+2)}+\frac{t-2}{2(3t+2)}$$

$$\frac{-2(t-2)}{6(3t+2)}+\frac{3(t-2)}{6(3t+2)}$$

$=$

$$\frac{-2(t-2)+3(t-2)}{6(3t+2)}$$

$=$

$$\frac{t-2}{6(3t+2)}$$

2. Add the rational expression, with different denominators.

$$\frac{2a}{4a^2-9b^2}+\frac{1}{2a-3b}$$

$=$

$$\frac{2a}{(2a+3b)(2a-3b)}+\frac{1}{2a-3b}$$

$=$

$$\frac{2a}{(2a+3b)(2a-3b)}+\frac{1\cdot(2a+3b)}{(2a+3b)(2a-3b)}$$

$=$

$$\frac{2a+2a+3b}{(2a+3b)(2a-3b)}$$

$=$

$$\frac{4a+3b}{(2a+3b)(2a-3b)}$$

3. Subtract the rational expressions.

$$\frac{6}{2a+2}-\frac{a+4}{4a-4}$$

$$\frac{6}{2(a+1)}-\frac{a+4}{4(a-1)}$$

$$\frac{2(a-1)6}{4(a-1)(a+1)}-\frac{(a+1)(a+4)}{4(a-1)(a+1)}$$

$=$

$$\frac{12a-12-(a^2+5a+5)}{4(a-1)(a+1)}$$

$=$

$$\frac{-a^2+7a-17}{4(a-1)(a+1)}$$

4. Subtract the rational expressions.

$$\frac{3a}{9a^2-4b^2}-\frac{1}{3a+2b}$$

$$\frac{3a}{(3a-2b)(3a+2b)}-\frac{1}{3a+2b}$$

$$\frac{3a}{(3a-2b)(3a+3b)}+\frac{-1\cdot(3a-2b)}{(3a+2b)(3a-2b)}$$

$=$

$$\frac{3a - 3a - 2b}{(3a+2b)(3a-2b)}$$

$=$

$$\frac{-2b}{(3a+2b)(3a-2b)}$$