

1 Complex Fractions 7.6

By the end of this section, you should be able to solve the following problems.

1. Simplify the complex fraction.

$$\frac{\frac{x-2}{x}}{\frac{x+3}{2}}$$

2. Simplify the complex fraction.

$$\frac{\frac{1}{x} - x}{\frac{x^2-1}{3}}$$

3. Simplify the complex fraction.

$$\frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{y^2} - \frac{1}{x^2}}$$

4. Simplify the complex fraction.

$$\frac{2 + \frac{1}{x}}{1 - \frac{2}{x}}$$

2 Concepts

A complex fraction is a multi-tiered expression that usually has fractions in the numerator and fractions in the denominator. The trick to simplifying

complex fractions is to combine all fractions or terms in the numerator and denominator into one fraction and then take the reciprocal of the denominator and multiply. As in our example below, we always will make the numerator and denominator into one fraction before inverting and multiplying.

2.1 Example

1. Simplify the complex fraction.

$$\frac{\frac{2}{x} + \frac{1}{y}}{\frac{1}{x^2} - \frac{1}{y^2}}$$

First, we write the fractions in the numerator and denominator with a common denominator

$$\frac{\frac{2y+x}{xy}}{\frac{y^2-x^2}{x^2y^2}}$$

Now we take the reciprocal of the denominator and multiply.

$$\frac{2y+x}{xy} \cdot \frac{x^2y^2}{y^2-x^2}$$

=

$$\frac{xy(2y+x)}{y^2-x^2}$$

2. Simplify the complex fraction.

$$\frac{2 + \frac{1}{r^2}}{3 - \frac{s}{r}}$$

Writing the numerator and denominator as one fraction we have.

$$\frac{\frac{2r^2+1}{r^2}}{\frac{3r-s}{r}}$$

Inverting and multiplying we get.

$$\frac{2r^2+1}{r^2} \cdot \frac{r}{3r-s}$$

=

$$\frac{2r^2+1}{3r^2-sr}$$

3 Facts

1. To simplify a complex fraction, combine all fractions in the numerator and denominator by using common denominators.
2. After the numerator and denominator of the complex fraction have been written as one fraction, invert the denominator and multiply.
3. Always remember to divide out any common factor in the numerator and denominator before multiplying.

4 Exercises

1. Simplify the complex fraction.

$$\frac{\frac{x-2}{x}}{\frac{x+3}{2}}$$

2. Simplify the complex fraction.

$$\frac{\frac{1-x^2}{x}}{\frac{x^2-1}{3}}$$

3. Simplify the complex fraction.

$$\frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{y^2} - \frac{1}{x^2}}$$

4. Simplify the complex fraction.

$$\frac{2 + \frac{1}{x}}{1 - \frac{2}{x}}$$

=

5 Solutions

1. Simplify the complex fraction.

$$\frac{\frac{x-2}{x}}{\frac{x+3}{2}}$$

=

$$\frac{x-2}{x} \cdot \frac{2}{x+3}$$
$$\frac{2x-4}{x^2+3x}$$

2. Simplify the complex fraction.

$$\frac{\frac{1-x^2}{x}}{\frac{x^2-1}{3}}$$
$$\frac{1-x^2}{x} \cdot \frac{3}{x^2-1}$$
$$\frac{(1+x)(1-x)}{x} \cdot \frac{3}{(x+1)(x-1)}$$
$$\frac{(1+x)(1-x)}{x} \cdot \frac{3}{-(1-x)(1+x)}$$

=

$$\frac{-3}{x}$$

3. Simplify the complex fraction.

$$\frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{y^2} - \frac{1}{x^2}}$$

$$\frac{y+x}{xy} \cdot \frac{x^2 y^2}{x^2 - y^2}$$

$$\frac{xy(y+x)}{(x+y)(x-y)}$$

$$\frac{xy}{(x-y)}$$

4. Simplify the complex fraction.

$$\frac{2 + \frac{1}{x}}{1 - \frac{2}{x}}$$

=

$$\frac{\frac{2x+1}{x}}{\frac{x-2}{x}}$$

$$\frac{2x+1}{x} \cdot \frac{x}{x-2}$$

$$\frac{2x+1}{x-2}$$