1 Equations With Rational Expressions 7.7

By the end of this section, you should be able to solve the following problems.

1. Solve the equation for the given unknown.

$$\frac{1}{3} - \frac{5}{x} = 2$$

2. Solve the equation for the given unknown.

$$\frac{4}{5x+1} = \frac{2}{2x-1}$$

3. Solve the equation for the given unknown.

$$\frac{2}{x-1} + \frac{x-2}{3} = \frac{4}{x-1}$$

4. Solve the equation for the given unknown.

$$\frac{2}{x} + \frac{1}{x+1} = \frac{3x^2-2x}{x^2+x}$$

2 Concepts

The goal, whenever we solve an equation with rational terms, is to get the equation all in a line. The way we do this is by getting rid of denominators by multiplying both sides by the lowest common denominator of all the terms.

Put bluntly, whenever we solve equations, we hate denominators. In the following example, we illustrate this process.

2.1 Examples

1. Solve for the given variable.

$$\frac{4}{x} + \frac{1}{2} = 4$$

The lowest common denominator of the rational terms in this expression is $2x$. So we multiply both sides by $2x$ and distribute automatically.

$$(2x)\frac{4}{x} + (2x)\frac{1}{2} = 4(2x)$$

Cancelling the denominations we get.

$$8 + x = 8x$$

$$-x \quad - x$$

$$\overline{8 = 7x}$$

$$\frac{8}{7} = \frac{7x}{7}$$

$$\frac{8}{7} = x$$

In our next example, we must factor denominators to find a common denominator.

2.

$$\frac{6x}{x^2-4}+\frac{x-1}{x+2}=3$$

First, we factor the denominator so we can find the least common denominator to multiply both sides by

$$\frac{6x}{(x+2)(x-2)}+\frac{x-1}{x+2}=3$$

Now we multiply both side by the Least Common Denominator.

$$\frac{(x+2)(x-2)6x}{(x+2)(x-2)}+(x+2)(x-2)\cdot\frac{(x-1)}{x+2}=3(x+2)(x-2)$$

=

$$6x+(x-2)(x-1)=3(x+2)(x-2)$$

=

$$6x+x^2-3x+2=3x^2-12$$

$$3x+x^2+2=3x^2-12$$

$$-3x-x^2-2 \quad -3x-x^2-2$$

$$\overline{0=2x^2-3x-14}$$

$$0 = 2x^2 - 3x - 14$$

$$0 = (2x^2 + 4x) + (-7x - 14)$$

$$0 - 2x(x + 2) - 7(x + 2)$$

$=$

$$0 = (x + 2)(2x - 7)$$

Finally, this equation will equal zero when either $x = -2$ or $x = \frac{7}{2}$.

In our last example, we have relatively prime denominators on either side of an equal sign.

3. Solve for x.

$$\frac{3}{x-1} = \frac{2x}{x+1}$$

Multiply both side by the common denominator.

$$\frac{(x-1)(x+1)3}{x-1} = \frac{(x-1)(x+1)2x}{x+1}$$

$$3(x + 1) = 2x(x - 1)$$

$$3x + 3 = 2x^2 - 2x$$

$$-3x - 3 \quad - 3x - 3$$

$$\overline{0 = 2x^2 - 5x - 3}$$

$$0 = 2x^2 - 6x + x - 3$$

$$0 = 2x(x - 3) + 1 \cdot (x - 3)$$

$$0 = (x - 3)(2x + 1)$$

Therefore, the roots of the equation are $x = 3$ and $x = \frac{-1}{2}$.

3 Facts

1. Whenever we go to solve equations with rational expressions in them, our goal is to write the expression all in one line.

2. To write the expression in a line, we multiply both sides of the equation by the lowest common denominator of the rational terms.

3. After multiplying both sides by the LCM, we distribute the LCM across all the terms.

4 Exercises

1. Solve the equation for the given unknown

$$\frac{1}{3} - \frac{5}{x} = 2$$

2. Solve the equation for the given unknown.

$$\frac{4}{5x-1} = \frac{2}{2x-1}$$

3. Solve the equation for the given unknown.

$$\frac{2}{x-1} + \frac{x-2}{3} = \frac{4}{x-1}$$

4. Solve the equation for the given unknown.

$$\frac{2}{x} + \frac{1}{x+1} = \frac{3x^2-2x}{x^2+x}$$

5 Solutions

1. Solve the equation for the given unknown

$$\frac{1}{3} - \frac{5}{x} = 2$$

$$\frac{(3x)\cdot 1}{3} - \frac{(3x)\cdot 5}{x} = 2(3x)$$

$$x - 15 = 6x$$

$$-x \quad - x$$

$$\overline{-15 = 5x}$$

$$\frac{-15}{5} = \frac{5x}{5}$$

$$-3 = x$$

2. Solve the equation for the given unknown.

$$\frac{4}{5x-1} = \frac{2}{2x-1}$$

$$\frac{(2x-1)(5x-1)4}{5x-1} = \frac{(2x-1)(5x-1)2}{2x-1}$$

$$8x - 4 = 10x - 2$$

$$-8x \quad - 8x$$

$$\overline{-4 = 2x - 2}$$

$$\begin{array}{rl} 2 & 2 \\ \hline -2 = 2x & \end{array}$$

$$\frac{-2}{2} = \frac{2x}{2}$$

$$-1 = x$$

3.

$$\frac{2}{x-1} + \frac{x-2}{3} = \frac{4}{x-1}$$

$$\frac{3(x-1)2}{x-1} + \frac{3(x-1)(x-2)}{3} = \frac{4 \cdot 3(x-1)}{x-1}$$

$$6 + x^2 - 3x + 2 = 12$$

$$x^2 - 3x + 8 = 12$$

$$\begin{array}{rl} -12 & -12 \\ \hline x^2 - 3x - 4 = 0 & \end{array}$$

$$(x+1)(x-4) = 0$$

The roots of the equation are $x = -1$ and $x = 4$.

4. Solve the equation for the given unknown.

$$\frac{2}{x} + \frac{1}{x+1} = \frac{3x^2 - 2x}{x^2 + x}$$

$$\frac{x(x+1)2}{x} + \frac{x(x+1)\cdot 1}{x+1} = \frac{x(x+1)(3x^2-2x)}{x(x+1)}$$

$$2x + 2 + x = 3x^2 - 2x$$

$$3x + 2 = 3x^2 - 2x$$

$$-3x - 2 \qquad -3x - 2$$

$$\overline{0 = 3x^2 - 5x - 2}$$

$$0 = 3x^2 - 6x + x - 2$$

$$0 = 3x(x-2) + 1\cdot(x-2)$$

$$0 = (x-2)(3x+1)$$

Therefore, the roots of the equation are $x = 2$ and $x = \frac{-1}{3}$