1 Finding Roots 8.1

By the end of this section, you should be able to solve the following problems.

1. Find the principal square root of the given number.

$$\sqrt{225}$$

2. Find the value of the radical.

$$\sqrt[5]{243}$$

3. Identify the number as rational, irrational, or non-real.

$$\sqrt[3]{\frac{27}{64}}$$

4. Mario is flying a kite with a string of 80 ft. The horizontal distance between Mario and the kite is 35 ft. How high is the kite above the ground if Mario's hands are 4 feet above the ground? Approximate the answer to the thousandths place.

2 Concepts

The square root of a number simply asks the question, "What factor when used twice gives that number back". The cube root of a number asks, "What number when used as a factor 3 times gives that number back." Therefore, for any n, the n^{th} root of a number asks, "What number when used as a factor n times will give that number back." Some simple examples will illustrate.

2.1 Examples

1. The $\sqrt{4} = 2$ because $2 \cdot 2 = 4$
2. The $\sqrt[3]{8} = 2$ because $2 \cdot 2 \cdot 2 = 8$
3. The $\sqrt[4]{\frac{16}{81}} = \frac{2}{3}$ because $\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{16}{81}$

3 Concepts

There are a few other facts to notice about finding roots. The first is that we are always solving the equation $x^n = a$, and, in particular, when we are solving $x^2 = a$, a could be either positive or negative. For example, in the statement $\sqrt{4}$ could have root $+2$ or -2 because $(+2)(+2) = 4$ or $(-2)(-2) = 4$. So for the solution to $x^2 = a$, we write $x = \pm\sqrt{a}$. An even more technical reason for this comes from the definition of absolute value. Absolute value is defined as the square root of a squared number. That is,

$\|x\| = \sqrt{x^2}$. Therefore,

$$x^2 = a$$

$$\sqrt{x^2} = \sqrt{a}$$

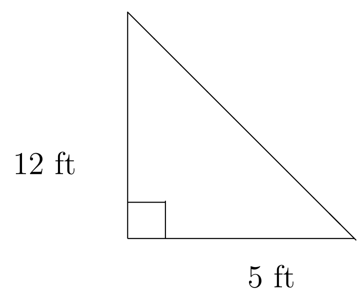
$$\|x\| = \sqrt{a}$$

$$x = \pm\sqrt{a}$$

Aside from what a principal square root is, we must also understand that it is not possible to take an even root of a negative number. For example, the $\sqrt{-4}$ does not exist in the real number system because no number when used a factor twice will give a negative product. Therefore, we say the solution of $\sqrt{-4}$ is not real. In general, $\sqrt{x}$ is only true for $x \geq 0$ In our next example, we use our ability to find square roots to solve an application of the Pythagorean Theorem.

3.1 Example

A tetherball pole cast a shadow 5ft long at a particular time of the day. If the tetherball pole is 12ft high, find the distance between the top of the pole and the tip of its shadow. Below is a picture of this situation.



We can assume that the pole makes a right angle with the ground because otherwise the problem won't work. Now that we imagine a right triangle, we can apply the Pythagorean Theorem. The a and b in the formula below designate the legs that meet at right angles. c always designates the longest side of the right angle which is the side opposite the right angle. It is called the hypotenuse.

We now state the Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Substituting we have:

$$(12)^2 + (5)^2 = c^2$$

$$144 + 25 = c^2$$

$$169 = c^2$$

$$\sqrt{169} = \sqrt{c^2}$$

$$13 = c$$

Notice, we take principal square root here because it is an applied problem

4 Facts

1. We take the positive square root of a number to be the principal square root. That is, the principle square root of $\pm\sqrt{a}$ is $+\sqrt{a}$.
2. Any even root of a negative number will not be real.
3. The square root of any prime number is always irrational. Irrationals can not be written in ratio form $\frac{a}{b}(b \neq 0)$.
4. Any real number that is not irrational is rational.
5. In the Pythagorean Theorem, the legs a and b meet at right angles, and the longest side c, is opposite the right angle. The formula for the Pythagorean Theorem is stated below

$$a^2 + b^2 = c^2$$

5 Exercises

1. Find the principal square root of the given number.

$$\sqrt{225}$$

2. Find the value of the radical

$$\sqrt[5]{243}$$

3. Identify the number as rational, irrational, or non-real.

$$\sqrt[3]{\frac{27}{64}}$$

4. Mario is flying a kite with a string of 80 feet. The horizontal distance between Mario and the kite is 35ft. How high is the kite above the ground if Mario's hands are 4 feet above the ground? Approximate the answer to the thousandths place.

6 Solutions

1. Find the principal square root of the given number.

$$\sqrt{225}$$

$$\sqrt{225} = 15$$

2. Find the value of the radical

$$\sqrt[5]{243}$$

$$\sqrt[5]{243} = 3$$

because

$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$$

3. Identify the number as rational, irrational, or non-real.

$$\sqrt[3]{\frac{27}{64}}$$

$$\sqrt[3]{\frac{27}{64}} = \frac{3}{4}$$

which is a rational number.

4. Mario is flying a kite with a string of 80 feet. The horizontal distance between Mario and the kite is 35ft. How high is the kite above the

ground if Mario's hands are 4 feet above the ground? Approximate the answer to the thousandths place.

$$(35)^2 + x^2 = (80)^2$$

$$1225 + x^2 = 6400$$

$$-1225 \quad\; - 1225$$

$$\overline{x^2 = 5175}$$

$$\sqrt{x^2} = \sqrt{5175}$$

$$x \approx 71.937$$

$$x + 4 \approx 75.937$$