

1 Multiplication and Division of Radicals 8.2

By the end of this section, you should be able to solve the following problems.

1. Use the product rule and simplify the expression. Assume that all variables are positive.

$$\sqrt{98x^5y^3} \cdot \sqrt{4x^2y^2}$$

2. Simplify the radical.

$$\sqrt{128}$$

3. Perform the indicated operations and simplify (the variables represent positive numbers).

$$\frac{\sqrt{15x^3y}}{\sqrt{3xy}}$$

4. Perform the indicated operations and simplify (the variables represent positive numbers).

$$\frac{\sqrt{12a^3b}}{\sqrt{24a^2b^2}}$$

2 Concepts

Multiplying radicals is related to our rules for exponents. When we multiply radicals, we rewrite radicals as rational powers and apply the rule $a^n b^n =$

$(ab)^n$. But in this section we will not go into that much detail. For us it is enough to simplify elements underneath the same radical.

2.1 Example

Multiply the radicals.

$$\sqrt{2} \cdot \sqrt{3}$$

=

$$\sqrt{6}$$

because

$$\sqrt{2} = 2^n$$

and

$$\sqrt{3} = 3^n$$

and

$$(2^n \cdot 3^n) = (2 \cdot 3)^n$$

Therefore, we have

$$(6)^n$$

Rewriting gives

$$\sqrt{6}$$

We will define n as a rational exponent is in a later section. The important thing to remember is that the power rules apply to radicals.

3 Concepts

When we divide radicals, again the fundamental power rules apply. The operative rule here is: $\frac{(a)^n}{(b)^n} = (\frac{a}{b})^n$. We define n as a rational exponent later.

3.1 Example

Simplify the quotient of radicals.

$$\begin{aligned} & \frac{\sqrt{15a^3b^4}}{\sqrt{3abn4}} \\ = & \\ & \sqrt{\frac{15a^3b^4}{3ab^3}} \\ = & \\ & \sqrt{5a^2b} \end{aligned}$$

Notice that we could rewrite.

$$\frac{\sqrt{15a^3b^4}}{\sqrt{3ab^4}}$$

as

$$\frac{(15a^3b^4)^n}{(3ab^4)^n}$$

=

$$\left(\frac{15a^3b^4}{3ab^4}\right)^n$$

=

$$(5a^2b)^n$$

Again, we will define n in a later section as a rational exponent.

4 Concepts

The most important thing to look for, when simplifying square roots, are factors of the radicand that are perfect squares. When simplify cube roots, we look for factors of the radicand that are perfect cubes. If we have a fourth root, we look for factors that are perfect fourth powers. Then once we identify perfect squares, cubes, fourths, etc., we can pull that base out from under the radical. We now turn to examples.

4.1 Example

Simplify the radical.

$$\sqrt{120}$$

Now we look for factors of 120 that are perfect squares. Since 4 divides 120, we can rewrite the problem above as,

$$\sqrt{4 \cdot 120}$$

=

$$\sqrt{4} \cdot \sqrt{30}$$

=

$$2\sqrt{30}$$

We pull the 2 out because the square root of 4 is 2.

5 Facts

1. We may multiply radicals that have the same root. To do this, take the product of the radicands and write them under the same radical.

2. Multiplication of radicals operate the same as the power rules. $(a^n b^n) = (ab)^n$
3. Division of radicals follows the power rule $\frac{(a)^n}{(b)^n} = \left(\frac{a}{b}\right)^n$
4. When simplifying radicals, always seek the largest perfect square, cube, etc. that evenly divides the radicand. Then pull out the square root, cube root, etc. out from the radical.

6 Exercises

1. Use the product rule and simplify the expression. Assume that all variables are positive.

$$\sqrt{98x^5y^3} \cdot \sqrt{4x^2y^2}$$

2. Simplify the radical.

$$\sqrt{128}$$

3. Perform the indicated operations and simplify (the variables represent positive numbers).

$$\frac{\sqrt{15x^3y}}{\sqrt{3xy}}$$

4. Perform the indicated operations and simplify (the variables represent positive numbers).

$$\frac{\sqrt{12a^3b}}{\sqrt{24a^2b^2}}$$

7 Solutions

1. Use the product rule and simplify the expression. Assume that all variables are positive.

$$\sqrt{98x^5y^3} \cdot \sqrt{4x^2y^2}$$

=

$$\sqrt{2 \cdot 2^2 \cdot 7^2 \cdot x^6 \cdot x \cdot y^4 \cdot y} =$$
$$14x^3y^2\sqrt{2xy}$$

2. Simplify the radical.

$$\sqrt{128} = \sqrt{64 \cdot 2} = 8\sqrt{2}$$

3. Perform the indicated operations and simplify (the variables represent positive numbers).

$$\frac{\sqrt{15x^3y}}{\sqrt{3xy}}$$

=

$$\sqrt{\frac{15x^3y}{3xy}}$$

=

$$\sqrt{5x^2} = x\sqrt{5}$$

4. Perform the indicated operations and simplify (the variables represent positive numbers).

$$\frac{\sqrt{12a^3b}}{\sqrt{24a^2b^2}}$$

=

$$\sqrt{\frac{12a^3b}{24a^2b^2}}$$

=

$$\sqrt{\frac{a}{2b}}$$