1 Addition and Substraction of Radicals 8.3

By the end of this section, you should be able to solve the following problems.

1. Perform the indicated operation.

$$4\sqrt[4]{3} - 8\sqrt[4]{3}$$

2. Perform the indicated operation.

$$\frac{1}{2}\sqrt{10} + \frac{2}{3}\sqrt{10}$$

3. Perform the indicated operation and simplify.

$$2\sqrt{3}(4\sqrt{27} - 5\sqrt{32})$$

4. Perform the indicated operation and simplify. Assume that the variables represent non-negative real numbers.

$$(2\sqrt{a} + \sqrt{b}) \cdot (5\sqrt{a} + 4\sqrt{b})$$

2 Concepts

The key thing to remember when adding or subtracting radical expressions is that the only time you can add or subtract is when the roots and radicand are the same. This is exactly the same idea we apply when we are adding or subtracting like terms. Below are two examples. In the first, we simply add because we have like radicals. In the second, we simplify the radicals before adding.

1. Simplify.

$$5\sqrt{3} + 4\sqrt{3}$$

=

$$9\sqrt{3}$$

2. Simplify.

$$8\sqrt{12} - 2\sqrt{48}$$

Write 12 and 48 as products of square factors.

$$8\sqrt{4 \cdot 3} - 2\sqrt{16 \cdot 3}$$

Since 4 and 16 are perfect square we can pull out a 2 and a 4, respectively, from under the radical.

$$8 \cdot 2\sqrt{3} - 2 \cdot 4\sqrt{3}$$

$$16\sqrt{3} - 8\sqrt{3}$$

$=$

$$8\sqrt{3}$$

In our next two examples, we apply the distributive law, and since we already know how to multiply radicals, this should present no difficulties.

2.1 Examples

1. Distribute.

$$2\sqrt{2}(\sqrt{54}+3\sqrt{18})$$

$$2\sqrt{2}(\sqrt{9\cdot 6}+3\sqrt{9\cdot 2})$$

$=$

$$2\sqrt{2}(3\sqrt{6}+3\cdot 3\sqrt{2})$$

$$2\sqrt{2}(3\sqrt{6}+9\sqrt{2})$$

$$2\cdot 3\sqrt{2\cdot 6}+2\cdot 9\sqrt{2\cdot 2}$$

$$6\sqrt{2\cdot 2\cdot 3}+18\sqrt{4}$$

$$6\sqrt{4\cdot 3}+18\sqrt{4}$$

$$12\sqrt{3}+36$$

2. Distribute.

$$(3\sqrt{x} + \sqrt{y}) \cdot (\sqrt{x} + 2\sqrt{y})$$

$$3x + 6\sqrt{xy} + \sqrt{xy} + 2y$$

$$3x + 7\sqrt{xy} + 2y$$

3 Facts

1. When adding or subtracting radicals, the root (or index) must be exactly the same, and the radicand must be exactly the same

2. Sometimes, if the radicands are not alike, they can be simplified, so that the same number is under the radical. Then the radicals can be added.

4 Exercises

1. Perform the indicated operations.

$$4\sqrt[4]{3} - 8\sqrt[4]{3}$$

2. Perform the indicated operations.

$$\frac{1}{2}\sqrt{10} + \frac{2}{3}\sqrt{10}$$

3. Perform the indicated operations and simply. Assume that the variables represent non-negative real numbers.

$$2\sqrt{3}(\sqrt{27} - 5\sqrt{32})$$

4. Perform the indicated operations and simply. Assume that the variables represent non-negative real numbers.

$$(2\sqrt{a} + \sqrt{b})(5\sqrt{a} + 4\sqrt{b})$$

5 Solutions

1. Perform the indicated operations.

$$4\sqrt[4]{3} - 8\sqrt[4]{3}$$

=

$$-4\sqrt[4]{3}$$

2. Perform the indicated operations.

$$\frac{1}{2}\sqrt{10} + \frac{2}{3}\sqrt{10}$$

$$\frac{1}{2} \cdot \frac{3}{3}\sqrt{10} + \frac{2}{3} \cdot \frac{2}{2}\sqrt{10}$$

=

$$\frac{3}{6}\sqrt{10} + \frac{4}{6}\sqrt{10}$$

=

$$\frac{7}{6}\sqrt{10}$$

3. Perform the indicated operations and simply. Assume that the variables represent non-negative real numbers.

$$2\sqrt{3}(\sqrt{27} - 5\sqrt{32})$$

$$2\sqrt{3}(\sqrt{9 \cdot 3} - 5\sqrt{16 \cdot 2})$$

$$2\sqrt{3}(3\sqrt{3} - 20\sqrt{2})$$

$$6\sqrt{9} - 40\sqrt{6}$$

$$18 - 40\sqrt{6}$$

4. Perform the indicated operations and simply. Assume that the variables represent non-negative real numbers.

$$(2\sqrt{a} + \sqrt{b})(5\sqrt{a} + 4\sqrt{b})$$

$$= 10a + 8\sqrt{ab} + 5\sqrt{ab} + 4b$$

$$= 10a + 13\sqrt{ab} + 4b$$