1 Rationalizing the Denominator 8.4

By the end of this section, you should be able to solve the following problems.

1. Rationalize the denominator and simplify.

$$\frac{3}{\sqrt{75}}$$

2. Rationalize the denominator and express the result in simplified form.

$$\sqrt{\frac{9}{2x}}$$

3. Rationalize the denominator and express the result in simplified form.

$$\frac{\sqrt{6ab^3}}{\sqrt{4a^3b}}$$

4. Rationalize the denominator and simplify.

$$\frac{2+\sqrt{3}}{2-\sqrt{3}}$$

2 Concepts

In this section, we learn a technique called rationalizing the denominator. The reason we do this is because it is useful to get all radicals out of the denominator. The way we do this is by multiplying both numerator and denominator by a factor that makes the denominator a perfect square. In our first example, we rationalize an expression that occurs frequently in trigonometry.

2.1 Example

Rationalize the denominator.

$$\frac{1}{\sqrt{2}}$$

=

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

=

$$\frac{\sqrt{2}}{\sqrt{4}}$$

=

$$\frac{\sqrt{2}}{2}$$

In our next two examples, we apply rules for exponents and rationalization to simplify rational expressions.

2.2 Examples

1. Simplify the radical expression and rationalize the denominator.

$$\frac{\sqrt{16xy}}{\sqrt{3x}}$$

$$\sqrt{\frac{16xy}{3x}}$$

$$\sqrt{\frac{16y}{3}}$$

$$\frac{4\sqrt{y}}{\sqrt{3}}$$

$$\frac{4\sqrt{y}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

=

$$\frac{2\sqrt{3y}}{3}$$

2. Simplify the radical expression and rationalize the denominator

$$\frac{\sqrt{24x^3y^2}}{\sqrt{16x^2y}}$$

=

$$\sqrt{\frac{24x^3y^2}{16x^2y}}$$

=

$$\sqrt{\frac{3xy}{2}}$$

$$\frac{\sqrt{3xy}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$\frac{\sqrt{6xy}}{2}$$

3 Concepts

In our next example, we must use another technique to rationalize the denominator. In this method, we multiply the numerator and denominator by what is called the conjugate of the denominator. If the denominator is of the form $a+b$. The conjugate will have the form $a-b$. Here we are applying the idea of the difference of two squares to eliminate radicals in the denominator. An example, of the conjugate of $1+\sqrt{3}$ is $1-\sqrt{+}3$ because, when multiplied, the square roots will be eliminated: $(1+\sqrt{3})(1-\sqrt{3}) = 1-3 = -2$. In our next example, we apply conjugates to a rational expression to rationalize the denominator.

3.1 Example

Rationalize the denominator.

$$\frac{5}{2+\sqrt{7}}$$

Now we multiply numerator and denominator by the conjugate of the denominator which is

$$\frac{5}{2+\sqrt{7}} \cdot \frac{2-\sqrt{7}}{2-\sqrt{7}}$$

=

$$\frac{5(2-\sqrt{7})}{4-7}$$

$$\frac{10-5\sqrt{7}}{-3}$$

4 Facts

1. When rationalizing a denominator, we always multiply both numerator and denominator by a radical expression that will make a perfect square, cube, etc. to eliminate the radical in the denominator.

2. Whenever we have a quotient of radical expressions or a radical of a quotient, the goal is always to simplify the expression as much as possible before rationalizing any denominators.

3. The conjugate of an expression of the form $a + b$ is $a - b$, and the conjugate of the expression $a - b$ is $a + b$. Conjugates are used to eliminate radicals by forming the difference of two squares.

5 Exercises

1. Rationalize the denominator and simplify.

$$\frac{3}{\sqrt{75}}$$

2. Rationalize the denominator and express the result in the simplified form.

$$\sqrt{\frac{9}{2x}}$$

3. Rationalize the denominator and express the result in the simplified form.

$$\frac{\sqrt{6ab^3}}{\sqrt{4a^3b}}$$

4. Rationalize the denominator and simplify

$$\frac{2+\sqrt{3}}{2-\sqrt{3}}$$

6 Solutions

1. Rationalize the denominator and simplify.

$$\frac{3}{\sqrt{75}}$$

=

$$\frac{3}{\sqrt{25 \cdot 3}}$$

=

$$\frac{3}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

2. Rationalize the denominator and express the result in the simplified form.

$$\sqrt{\frac{9}{2x}}$$

=

$$\frac{3}{\sqrt{2x}}$$

=

$$\frac{3}{\sqrt{2x}} \cdot \frac{\sqrt{2x}}{\sqrt{2x}}$$

=

$$\frac{3\sqrt{2x}}{2x}$$

3. Rationalize the denominator and express the result in the simplified form.

$$\frac{\sqrt{6ab^3}}{\sqrt{4a^3b}}$$
$$\sqrt{\frac{6ab^3}{4a^3b}}$$
$$\sqrt{\frac{3b^2}{2a^2}}$$
$$\frac{b\sqrt{3}}{a\sqrt{2}}$$
$$\frac{b\sqrt{3}}{a\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$\frac{b\sqrt{6}}{2a}$$

4. Rationalize the denominator and simplify

$$\frac{2+\sqrt{3}}{2-\sqrt{3}}$$

$$\frac{2+\sqrt{3}}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

=

$$\frac{(2+\sqrt{3})^2}{4-3}$$

=

$$(2+\sqrt{3})^2$$