

# 1 Fractional Exponents 8.5

By the end of this section, you should be able to solve the following problems.

1. Express in radical form and simplify

$$64^{\frac{2}{3}}$$

2. Simplify the expression. (Assume that all variables are positive).

$$(8x^2y)^{\frac{-1}{3}}$$

3. Simplify the expression. (Assume that all variables are positive).

$$\left(\frac{4p^3q}{9pq}\right)^{\frac{1}{2}}$$

4. Simplify the expression. (Assume that all variables are positive).

$$\frac{x^{\frac{2}{5}} \cdot x^{\frac{-7}{5}}}{x^{\frac{1}{3}} \cdot x^{\frac{-1}{3}}}$$

## 2 Concepts

In section 8.2 we said that the rules of exponents apply to radicals, now in section 8.5 we learn exactly why that is so. Actually, any radical expression can be rewritten as an exponential expression. By definition we write,

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

For example,

$$\sqrt[3]{x^2} = x^{\frac{2}{3}}$$

When we have square roots, we have an invisible power of 1 and an invisible root of two.  $\sqrt{a}$  actually means  $\sqrt[2]{a^1}$ . This becomes clear when we rewrite one radical in exponent form,  $a^{\frac{1}{2}}$ . Often, we have to simplify radicals when they are taken out of exponent form. The next example will show this

## 2.1 Example

Write as a radical in simplified form.

$$54^{\frac{2}{3}} = \sqrt[3]{54^2}$$

=

$$\sqrt[3]{54} \cdot \sqrt[3]{54} = \sqrt[3]{2 \cdot 27 \cdot 2 \cdot 27}$$

=

$$\sqrt[3]{27 \cdot 27 \cdot 2 \cdot 2}$$

=

$$\sqrt[3]{27} \cdot \sqrt[3]{27} \cdot \sqrt[3]{4}$$

=

$$9\sqrt[3]{4}$$

In the next three examples, we use rules of exponents to simplify as much as possible, and then rationalize denominators when necessary

## 2.2 Examples

1. Simplify

$$(98x^3y^5)^{\frac{1}{2}}$$

=

$$(2 \cdot 49 \cdot x^2 \cdot x \cdot y^4 \cdot y)^{\frac{1}{2}}$$

=

$$2^{\frac{1}{2}} \cdot 7 \cdot x \cdot x^{\frac{1}{2}} \cdot y^2 \cdot y^{\frac{1}{2}}$$

=

$$7xy^2\sqrt{2xy}$$

2. Simplify

$$\left(\frac{18x^3y}{5xy^4}\right)^{\frac{1}{2}}$$

$$\left(\frac{18x^2}{5y^3}\right)^{\frac{1}{2}}$$

$$\frac{2^{\frac{1}{2}} \cdot 3 \cdot x}{5^{\frac{1}{2}} \cdot y \cdot y^{\frac{1}{2}}}$$

$$\frac{3x\sqrt{2}}{y\sqrt{5y}} \cdot \frac{\sqrt{5y}}{\sqrt{5y}}$$

$$\frac{3x\sqrt{10y}}{5y^2}$$

3. Rewrite the expression in simplest radical form.

$$\frac{x^{\frac{2}{3}} \cdot x^{\frac{1}{2}}}{x^{\frac{3}{2}} \cdot x}$$

$$\frac{x^{\frac{4}{6}} \cdot x^{\frac{3}{6}}}{x^{\frac{3}{2}} \cdot x^{\frac{2}{2}}}$$

=

$$\frac{x^{\frac{7}{6}}}{x^{\frac{5}{2}}}$$

=

$$x^{\frac{7}{6}} \cdot x^{-\frac{5}{2}} = x^{\frac{7}{6}} \cdot x^{-\frac{15}{6}} = x^{-\frac{8}{6}}$$

$$x^{-\frac{8}{6}} = x^{-\frac{4}{3}} = \frac{1}{\sqrt[3]{x^4}} \cdot \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$$

$$\frac{\sqrt[3]{x^2}}{\sqrt[3]{x^6}} = \frac{\sqrt[3]{x^2}}{x^2}$$

### 3 Facts

1. The radical expression  $\sqrt[n]{a^m}$  is equivalent to the exponential expression  $a^{\frac{m}{n}}$ .
2. Any radical expression can be rewritten using fractional exponents and then simplified using the laws of exponents.

3. Once an expression has been simplified using the laws of exponents, it can be rewritten as a radical and simplified further or rationalized.
4. In general, all radical and exponential expressions must be simplified and/or rationalized when presented as an answer.

## 4 Exercises

1. Write the expression in radical form and simplify.

$$64^{\frac{2}{3}}$$

2. Simplify the expression. Assume that all variables are positive.

$$(8x^2y)^{\frac{-1}{3}}$$

3. Simplify the expression. Assume that all variables are positive.

$$\left(\frac{4p^3q}{9pq}\right)^{\frac{1}{2}}$$

4. Simplify the expression. Assume that all variables are positive.

$$\frac{x^{\frac{2}{5}} \cdot x^{\frac{-7}{5}}}{x^{\frac{4}{3}} \cdot x^{\frac{-1}{3}}}$$

## 5 Solutions

1. Write the expression in radical form and simplify.

$$\begin{aligned} & 64^{\frac{2}{3}} \\ & \sqrt[3]{64^2} \\ & = \\ & \sqrt[3]{64 \cdot 64} \\ & \sqrt[3]{64} \cdot \sqrt[3]{64} = 4 \cdot 4 = 16 \end{aligned}$$

2. Simplify the expression. Assume that all variables are positive.

$$\begin{aligned} & (8x^2y)^{-\frac{1}{3}} \\ & (8^{-\frac{1}{3}} \cdot x^{\frac{-2}{3}} \cdot y^{\frac{1}{3}}) \\ & \frac{\sqrt{y}}{2\sqrt[3]{x^2}} \cdot \frac{\sqrt[3]{x}}{\sqrt[3]{x}} \\ & \frac{\sqrt{y}\sqrt[3]{x}}{2x} \end{aligned}$$

3. Simplify the expression. Assume that all variables are positive.

$$\begin{aligned} & \left(\frac{4p^3q}{9pq}\right)^{\frac{1}{2}} \\ & \left(\frac{4p^2}{9}\right)^{\frac{1}{2}} \end{aligned}$$

$$\frac{\sqrt{4p^2}}{\sqrt{9}}$$
$$\frac{2p}{3}$$

4. Simplify the expression. Assume that all variables are positive.

$$\frac{x^{\frac{2}{5}} \cdot x^{\frac{-7}{5}}}{x^{\frac{4}{3}} \cdot x^{\frac{-1}{3}}}$$
$$\frac{x^{\frac{-5}{5}}}{x}$$

=

$$\frac{1}{x^2}$$