1 Equations with Radicals 8.6

By the end of this section, you should be able to solve the following problems.

1. Solve the equation.

$$\sqrt{3x-2} = 4$$

2. Solve the equation.

$$\sqrt{2x+3} = 2$$

3. Solve the equation.

$$\sqrt{x-3} = \sqrt{x} + 3$$

4. Solve the equation.

$$\sqrt{x} = \sqrt{2x+2} + 3$$

2 Concepts

The key to solving equations with radicals in them is to isolate a radical expression on one side of the equation and then square both sides to eliminate the radical. Whenever we solve equations with radicals in them, we must always check the answer. Many times we will find that a result does not satisfy the original equation because the original equation was not true, but our solution method assumes that it is. Solving an equation with radicals always employs the the following implication:

$$If\ a = b,\ then\ a^n = b^n$$

The problem is that the converse of this statement may not hold. That is,

$$If\ a^n = b^n,\ then\ a = b$$

And we are always assuming the converse is true whenever we raise each side to a power. This is a dangerous assumption. For example, $2 \neq -2$ But when solving a radical equation we assume $a = b$ so we could assume something like $2 = -2$. Proceeding, we could square both side and get $(2)^2 = (-2)^2$ which gives $4 = 4$ which is true. Unless we check our result we could generate a completely erroneous answer. Now we solve some examples keeping in mind that we must always check our result.

2.1 Example

Solve for x. First we isolate the radical by adding -5 to both sides.

$$x = \sqrt{x+7} + 5$$

$$-5 \quad -5$$

$$\overline{x - 5 = \sqrt{x+7}}$$

Squaring both sides.

$$(x-5)^2 = (\sqrt{x+7})^2$$

$$(x-5)(x-5) = x+7$$

$$x^2 - 10x + 25 = x + 7$$

$$-x - 7 \quad -x - 7$$

$$\overline{x^2 - 11x + 18 = 0}$$

$$(x-9)(x-2) = 0$$

Therefore, $x = 9$ or $x = 2$. Now we check the result. Substituting 2 in for x in the original equation, we have:

$$(2) = \sqrt{(2)+7} + 5$$

$$2 \neq 3 + 5$$

Therefore, 2 is not a solution. Substituting 9 in for x in the original equation, we have:

$$(9) = \sqrt{(9)+7} + 5$$

$$9 = 4 + 5$$

Therefore, 9 is the only solution to this equation. In our next example, it is necessary to square both sides twice to eliminate all radicals. Solve for x.

$$\sqrt{5x-1} - \sqrt{x+2} = 1$$

First we isolate one of the radicals.

$$\sqrt{5x-1} - \sqrt{x+2} = 1$$

$$+\sqrt{x+2} \qquad +\sqrt{x+2}$$

$$\overline{\sqrt{5x-1} = 1 + \sqrt{x+2}}$$

$$(\sqrt{5x-1})^2 = (1+\sqrt{x+2})^2$$

$$5x - 1 = (1+\sqrt{x+2})(1+\sqrt{x+2})$$

$$5x - 1 = 1 + 2\sqrt{x+2} + x + 2$$

$$x - 1 = 3 + x + 2\sqrt{x+2}$$

$$-x - 3 \qquad -x - 3$$

$$\overline{4x - 4 = 2\sqrt{x+2}}$$

$$\frac{4x-4}{2} = \frac{2\sqrt{x+2}}{2}$$

$$\frac{2(2x-2)}{2} = \frac{2\sqrt{x+2}}{2}$$

$$2x - 2 = \sqrt{x+2}$$

$$(2x-2)^2 = (\sqrt{x+2})^2$$

$$4x^2 - 8x + 4 = x + 2$$

$$-x-2 \qquad -x-2$$

$$\overline{4x^2 - 9x + 2 = 0}$$

$$(4x^2 - 8x) + (-x + 2) = 0$$

$$4x(x-2) - 1 \cdot (x-2) = 0$$

$$(x-2)(4x-1) = 0$$

Therefore, $x = 2$ or $x = \frac{1}{4}$ Check:

$$\sqrt{5(2)-1} - \sqrt{(2)+2} = 1$$

$$\sqrt{9} - \sqrt{4} = 1$$

$$3 - 2 = 1$$

Therefore, $x = 2$ is a solution to the equation.

$$\sqrt{5(\frac{1}{4})-1} - \sqrt{(\frac{1}{4})+2} = 1$$

$$\sqrt{\frac{5}{4}-\frac{4}{4}} - \sqrt{(\frac{1}{4})+\frac{8}{4}} = 1$$

$$\sqrt{\frac{1}{4}} - \sqrt{\frac{9}{4}} = 1$$

$$\frac{1}{2} - \frac{3}{2} = 1$$

$$\frac{-2}{2} = 1$$

Since $-1 \neq 1$, $\frac{1}{4}$ is not a solution to the equation. $\frac{1}{4}$ is an extraneous root.

3 Facts

1. Whenever we simplify equations with radicals in them, we must isolate a radical on one side of the equation and and square both sides. Sometimes we have to repeat this procedure more than once.

2. Whenever we solve radical equations, we use the relation $If\ a = b,\ then\ a^n = b^n$, and we must always check our results because our methods assume the converse, $If\ a^n = b^n,\ then\ a = b$, is true when it may not be.

3. Whenever we solve radical equations, we must check all our solutions because our methods sometimes produce extraneous roots.

4 Exercises

1. Solve the equation.

$$\sqrt{3x-2}=4$$

2. Solve the equation.

$$\sqrt{2x+3}=2$$

3. Solve the equation.

$$\sqrt{x-3}=\sqrt{x}+3$$

4. Solve the equation.

$$\sqrt{x}=\sqrt{2x+2}+3$$

5 Solutions

1. Solve the equation.

$$\sqrt{3x-2} = 4$$

$$(\sqrt{3x-2})^2 = (4)^2$$

$$3x - 2 = 16$$

$$+2 \quad +2$$

$$\overline{3x = 18}$$

$$\frac{3x}{3} = \frac{18}{3}$$

$$x = 6$$

2. Solve the equation.

$$\sqrt{2x+3} = 2$$

$$(\sqrt{2x+3})^2 = (2)^2$$

$$2x + 3 = 4$$

$$-3 \quad -3$$

$$\overline{2x = 1}$$

$$\frac{2x}{2} = \frac{1}{2}$$

$$x = \frac{1}{2}$$

3. Solve the equation.

$$\sqrt{x-3} = \sqrt{x} + 3$$

$$(\sqrt{x-3})^2 = (\sqrt{x} + 3)^2$$

$$x - 3 = x + 6\sqrt{x} + 9$$

$$-x - 9 \quad - x - 9$$

$$\overline{-12 = 6\sqrt{x}}$$

$$\frac{-12}{6} = \frac{6\sqrt{x}}{6}$$

$$-2 = \sqrt{x}$$

The square root of a number cannot be negative; therefore, this equation has no solution.

4. Solve the equation.

$$\sqrt{x} = \sqrt{2x+2} + 3$$

$$-3 \quad - 3$$

$$\overline{\sqrt{x} - 3 = \sqrt{2x+2}}$$

$$(\sqrt{x} - 3)^2 = (\sqrt{2x+2})^2$$

$$x - 6\sqrt{x} + 9 = 2x + 2$$

$$-x - 9 \qquad - x - 9$$

$$\overline{-6\sqrt{x} = x - 7}$$

$$(-6\sqrt{x})^2 = (x - 7)^2$$

$$36x = x^2 - 14x + 49$$

$$-36x \qquad - 36x$$

$$\overline{0 = x^2 - 50x + 49}$$

$$0 = (x - 1)(x - 49)$$

Neither solution is valid. This equation has no solution.