1 Square Root Property 9.1

By the end of this section, you should be able to solve the following problems.

1. Solve the equation.

$$4x^2 + 8 = 24$$

2. Solve the equation.

$$(2x - 5)^2 = 0$$

3. Solve the equation.

$$(2t + 7)^2 + 11 = 12$$

4. Solve the equation.

$$\frac{1}{4}(x - 2)^2 = 3$$

2 Concepts

When we solve the equation $x^2 = a$ we must be careful to recognize that there are two roots: a positive root and a negative root. So we have:

$$\sqrt{x^2} = \sqrt{a}$$

By definition of absolute value we get:

$$| x |= \sqrt{a}$$

Simplifying:

$$x = \pm\sqrt{a}$$

Both positive and negative $\sqrt{a}$ makes the statement $x^2 = a$ true. This is an important fact that we must always remember when solving equations where we must take square roots at the end. Below we provide some examples of the type of problems you will need to be able to solve.

2.1 Examples

1. Solve the equation.

$$3x^2 - 8 = -4$$

First we isolate the term with the square in it by adding +8 to both sides.

$$3x^2 = 4$$

Dividing both sides by 3 we have.

$$x^2 = \frac{4}{3}$$

Taking square roots of both sides.

$$\sqrt{x^2} = \sqrt{\frac{4}{3}}$$

$$| \, x \, | = \sqrt{\frac{4}{3}}$$

$$x = \pm\sqrt{\frac{4}{3}}$$

In the next example, we isolate the square of a binomial before the taking the square root of both sides.

2. Solve the equation.

$$(4x + 1)^2 + 4 = 13$$

Adding -4 to both sides we have.

$$(4x + 1)^2 = 9$$

Taking square roots of both sides we have:

$$\sqrt{(4x + 1)^2} = \sqrt{9}$$

$$| \, 4x + 1 \, | = 3$$

$$4x + 1 = \pm 3$$

Adding -1 to both sides and dividing by both sides by 4, we have:

$$x = \frac{-1 \pm 3}{4}$$

Simplifying we have the following two roots: $x = -1$ or $x = \frac{1}{2}$

3 Facts

1. When solving the equation $x^2 = a$. There are always two roots. Taking square roots of both sides we have: $x = \sqrt{a}$ or $x = -\sqrt{a}$

2. If the value of a in the equation $x^2 = a$ is negative, then the solution is non-real. For example, the equation $x^2 = -2$, has no real number solution because there is no such number that when squared gives -2.

4 Exercises

1. Solve the equation.

$$4x^2 + 8 = 24$$

2. Solve the equation.

$$(2x - 5)^2 = 0$$

3. Solve the equation.

$$(2t + 7)^2 + 11 = 12$$

4. Solve the equation.

$$\frac{1}{4}(x - 2)^2 = 3$$

5 Solutions

1. Solve the equation.

$$4x^2 + 8 = 24$$

$$4x^2 = 16$$

$$x^2 = 4$$

$$\sqrt{x^2} = \sqrt{4}$$

$$| \, x \, | = \sqrt{4}$$

$$x = \pm 2$$

2. Solve the equation.

$$(2x-5)^2 = 0$$

$$\sqrt{(2x-5)^2} = \sqrt{0}$$

$$|\ 2x-5\ | = \sqrt{0}$$

$$2x-5 = \pm 0$$

$$2x-5 = 0$$

$$2x = 5$$

$$x = \frac{5}{2}$$

3. Solve the equation.

$$(2t+7)^2 + 11 = 12$$

$$(2t+7)^2 = 1$$

$$\sqrt{(2t+7)^2} = \sqrt{1}$$

$$|\ 2t+7\ | = 1$$

$$2t+7 = \pm 1$$

$$2t = -7 \pm 1$$

$$t = \frac{-7 \pm 1}{2}$$

4. Solve the equation.

$$\frac{1}{4}(x-2)^2 = 3$$

$$(4)\frac{1}{4}(x-2)^2 = (4)3$$

$$(x-2)^2 = 12$$

$$\sqrt{(x-2)^2} = \sqrt{12}$$

$$|\, x-2 \,| = 2\sqrt{3}$$

$$x-2 = \pm 2\sqrt{3}$$

$$x = \pm 2\sqrt{3} + 2$$