1 The Quadratic Formula 9.3

By the end of this section, you should be able to solve the following problems.

1. Use the quadratic formula to solve the equation.

   \[ t^2 + 5t = 4 \]

2. Use the quadratic formula to solve the equation.

   \[ 5x^2 + 7x - 2 = 0 \]

3. Use the quadratic formula to solve the equation.

   \[ 2x^2 - 5x + 2 = 0 \]

4. Solve the application problem.

   In a group of children, each child gives a gift to every other child. If the number of gifts is 132, find the number of children.

2 Concepts

A quadratic equation is an equation in one variable where the highest integer power of the variable is 2. The general quadratic equation written in standard
The form is:

\[ ax^2 + bx + c = 0 \]

The solution of this equation can be derived by algebraic methods and it is:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

The way we use this equation is to arrange a specific example in standard form, and then plug the coefficients into the equation above. Below we will solve a quadratic equation and an application of the quadratic equation.

3 Example

1. Solve the quadratic equation using the quadratic formula.

\[ 3x^2 - x = 10 \]

First we put the equation in standard form by adding -10 to both sides to get:

\[ 3x^2 - x - 10 = 0 \]

The coefficient of the quadratic term is always a. Here, a=3. The coefficient of the linear term, x, is always b. Here, b=1. The constant
term is always c. Here, c=-10. We now proceed to solve this equation by substituting the coefficients into the equation.

\[
\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-10)}}{2(3)}
\]

\[
x = \frac{1 \pm \sqrt{1 + 120}}{6}
\]

Therefore, \(x = 2\) or \(x = \frac{-5}{3}\)

In our next example, we solve an application problem.

2. An abstract counting procedures adds one to a positive number and subtracts one from the same number and finds the product of the two numbers. If the product is 63, find the number.

Let \(x=\)the number. Then \((x + 1)(x - 1)\) will be the product of one less and one greater than the number. So our equation will be.

\[(x + 1)(x - 1) = 63\]
Multiplying we have:

\[ x^2 - 1 = 63 \]

Add +1 to both sides:

\[ x^2 = 64 \]

Taking square roots:

\[ \sqrt{x^2} = \pm \sqrt{64} \]

\[ x = \pm 8 \]

Since the number must be positive, we have \( x = 8 \).

4 Facts

1. The general quadratic equation in standard form is written.

\[ ax^2 + bx + c = 0 \]

2. To solve a quadratic equation simply write the equation in standard form and substitute the coefficients into the general solution.

3. The solution to the quadratic equation is written:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
5 Exercises

1. Use the quadratic formula to solve the equation.

\[ t^2 + 5t - 4 = 0 \]

2. Use the quadratic formula to solve the equation.

\[ 5x^2 + 7x - 2 = 0 \]

3. Use the quadratic formula to solve the equation.

\[ 2x^2 - 5x + 2 = 0 \]

4. In a group of children, each child gives a gift to every other child. If the number of gifts is 132, find the number of children.
6 Solutions

1. Use the quadratic formula to solve the equation.

\[ t^2 + 5t - 4 = 0 \]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-4)}}{2(1)}
\]
\[
x = \frac{-5 \pm \sqrt{41}}{2}
\]

2. Use the quadratic formula to solve the equation.

\[ 5x^2 + 7x - 2 = 0 \]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
x = \frac{-(7) \pm \sqrt{(7)^2 - 4(5)(-2)}}{2(5)}
\]
\[
x = \frac{-7 \pm \sqrt{89}}{10}
\]

3. Use the quadratic formula to solve the equation.

\[ 2x^2 - 5x + 2 = 0 \]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
x = \frac{-(5) \pm \sqrt{(-5)^2 - 4(2)(2)}}{2(2)}
\]
\[
x = \frac{5 \pm \sqrt{9}}{4}
\]
4. In a group of children, each child gives a gift to every other child. If 
the number of gifts is 132, find the number of children.

Let \( x \) be the number of children. Since each child must give a gift to 
every other child, each child must give \( x - 1 \) gifts. Then the following 
equation will count the number of gifts.

\[
x(x - 1) = 132
\]

\[
x^2 - x - 132 = 0
\]

\[
x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-132)}}{2(1)}
\]

\[
x = \frac{1 \pm \sqrt{1 + 528}}{2}
\]

\[
x = \frac{1 \pm 23}{2}
\]

Therefore, \( x = 12 \) or \( x = -11 \)