1 The Quadratic Formula 9.3

By the end of this section, you should be able to solve the following problems.

1. Use the quadratic formula to solve the equation.

$$t^2 + 5t = 4$$

2. Use the quadratic formula to solve the equation.

$$5x^2 + 7x - 2 = 0$$

3. Use the quadratic formula to solve the equation.

$$2x^2 - 5x + 2 = 0$$

4. Solve the application problem.

   In a group of children, each child gives a gift to every other child, If the number of gifts is 132, find the number of children.

2 Concepts

A quadratic equation is an equation in one variable where the highest integer power of the variable is 2. The general quadratic equation written in standard

form is:

$$ax^2 + bx + c = 0$$

The solution of this equation can be derived by algebraic methods and it is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The way we use this equation is to arrange a specific example in standard form, and then plug the coefficients into the equation above. Below we will solve a quadratic equation and an application of the quadratic equation.

3 Example

1. Solve the quadratic equation using the quadratic formula.

$$3x^2 - x = 10$$

First we put the equation in standard form by adding -10 to both sides to get:

$$3x^2 - x - 10 = 0$$

The coefficient of the quadratic term is always a. Here, a=3. The coefficient of the linear term, x, is always b. Here, b=1. The constant

term is always c. Here, c=-10. We now proceed to solve this equation by substituting the coefficients into the equation.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

x=

$$\frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-10)}}{2(3)}$$

$$\frac{1 \pm \sqrt{1 + 120}}{6}$$

x=

$$\frac{1 \pm 11}{6}$$

Therefore, $x = 2$ or $x = \frac{-5}{3}$

In our next example, we solve an application problem.

2. An abstract counting procedures adds one to a positive number and subtracts one from the same number and finds the product of the two numbers. If the product is 63, find the number.

   Let x=the number. Then $(x+1)(x-1)$ will be the product of one less and one greater than the number. So our equation will be.

$$(x+1)(x-1) = 63$$

Multiplying we have:

$$x^2 - 1 = 63$$

Add +1 to both sides:

$$x^2 = 64$$

Taking square roots:

$$\sqrt{x^2} = \pm\sqrt{64}$$

$$x = \pm 8$$

Since the number must be positive, we have $x = 8$.

4 Facts

1. The general quadratic equation in standard form is written.

$$ax^2 + bx + c = 0$$

2. To solve a quadratic equation simply write the equation in standard form and substitute the coefficients into the general solution.

3. The solution to the quadratic equation is written:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

5 Exercises

1. Use the quadratic formula to solve the equation.

$$t^2 + 5t - 4 = 0$$

2. Use the quadratic formula to solve the equation.

$$5x^2 + 7x - 2 = 0$$

3. Use the quadratic formula to solve the equation.

$$2x^2 - 5x + 2 = 0$$

4. In a group of children, each child gives a gift to every other child. If the number of gifts is 132, find the number of children.

6 Solutions

1. Use the quadratic formula to solve the equation.

$$t^2 + 5t - 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{41}}{2}$$

2. Use the quadratic formula to solve the equation.

$$5x^2 + 7x - 2 = 0$$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(5)(-2)}}{2(5)}$$

$$x = \frac{-7 \pm \sqrt{89}}{10}$$

3. Use the quadratic formula to solve the equation.

$$2x^2 - 5x + 2 = 0$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(2)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{9}}{4}$$

$$x = \frac{5 \pm 3}{4}$$

$$x = \frac{8}{4} = 2$$

$$x = \frac{2}{4} = \frac{1}{2}$$

4. In a group of children, each child gives a gift to every other child. If the number of gifts is 132, find the number of children.

   Let x be the number of children. Since each child must give a gift to every other child, each child must give $x - 1$ gifts. Then the following equation will count the number of gifts.

$$x(x-1) = 132$$

$$x^2 - x - 132 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-132)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1 + 528}}{2}$$

$$x = \frac{1 \pm 23}{2}$$

   Therefore, $x = 12$ or $x = -11$