MAT096 Review Workbook
Part II

Prof. McCormack

April 5, 2007
1 Complex Fraction

1.1 Example 1: Numerator Fractions

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 6. Complex Fractions in the MAT096 Review website, and watch the streaming video for 1) Numerator fractions. Check your work with the answers on the following page.

1. Simplify:
   \[
   \frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a}}
   \]

2. Simplify:
   \[
   \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{4}}
   \]

3. Simplify:
   \[
   \frac{\frac{2}{x} + \frac{1}{y}}{\frac{4}{x}}
   \]
1.2 Answers to Example 1: Numerator fractions

1. Simplify: \( \frac{\frac{1}{a} \cdot \frac{1}{b}}{\frac{1}{a} + \frac{1}{b}} \)

   \[ \frac{(\frac{a}{b})^{\frac{b}{a} + \frac{1}{a}(\frac{a}{b})}}{\frac{1}{a} + \frac{1}{b}} \frac{b \cdot a}{\frac{a}{b} \cdot \frac{a}{b}} \frac{b}{a} \cdot \frac{a}{1} \]

   Answer:

   \[ \frac{b}{a} + \frac{1}{b} \]

2. Simplify: \( \frac{\frac{1}{3} + \frac{1}{2}}{\frac{1}{3}} \)

   \[ \frac{(\frac{2}{3})^{\frac{2}{3} + \frac{1}{2}(\frac{2}{3})}}{\frac{3}{2}} \frac{\frac{3}{2} \cdot 3}{\frac{3}{2} \cdot 3} \frac{1}{\frac{3}{2}} \]

   Answer:

   \[ \frac{3}{2} + \frac{1}{2} \]

3. Simplify: \( \frac{\frac{2}{3} + \frac{1}{2}}{} \)

   \[ \frac{(\frac{2}{3})^{\frac{2}{3} + \frac{1}{2}(\frac{2}{3})}}{\frac{2}{3} + \frac{1}{2}} \frac{2 \cdot \frac{2}{3} \cdot \frac{3}{2}}{\frac{2}{3} \cdot \frac{3}{2}} \]

   Answer:
\[
\frac{2y + x}{xy} \cdot \frac{y}{3} = \frac{2y + x}{3y}
\]
1.3 Example 2: Constant in denominator

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 6. Complex Fractions in the MAT096 Review website, and watch the streaming video for 2) Constant in denominator. Check your work with the answers on the following page.

1. Simplify:
\[ \frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x} + 1} \]

2. Simplify:
\[ \frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x} + 1} \]

3. Simplify:
\[ \frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x} + 2} \]
1.4 Answers to Example 2: Constant in denominator

1. Simplify:
   \[ \frac{\frac{1}{x} + \frac{1}{y}}{x + 1} \]

   Answer:
   \[ \frac{y}{x} + \frac{x}{y} \]

2. Simplify:
   \[ \frac{\frac{1}{x} + \frac{1}{y}}{x + 1} \]

   Answer:
   \[ \frac{3}{2} + \frac{2}{3} \cdot \frac{6}{5} \]

3. Simplify:
   \[ \frac{\frac{1}{x} + \frac{1}{y}}{x + 1} \]

   Answer:
   \[ \frac{\frac{1}{x} + \frac{1}{y}}{x + 1} \]
\[
\frac{2y + x}{xy} \cdot \frac{x}{3 + 2x} \\
\frac{2y + x}{3y + 2xy}
\]
1.5 Example 3: Constant in numerator

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 6. Complex Fractions in the MAT096 Review website, and watch the streaming video for 3) Constant in numerator. Check your work with the answers on the following page.

1. Simplify:
\[
\frac{1 + \frac{1}{a}}{\frac{1}{b}}
\]

2. Simplify:
\[
\frac{2 + \frac{1}{x}}{\frac{1}{y}}
\]

3. Simplify:
\[
\frac{1 - \frac{a}{z}}{\frac{1}{z}}
\]
1.6 Answers to Example 2: Constant in denominator

1. Simplify:
\[
\frac{1 + \frac{1}{a}}{b}
\]

Answer:
\[
\left(\frac{a}{b}\right)\frac{1 + \frac{1}{a}}{b}
\]
\[
\frac{a}{b} + \frac{1}{b}
\]
\[
\frac{a + 1}{b}
\]
\[
\frac{a + 1}{b} \cdot \frac{b}{a}
\]
\[
\frac{a + b}{ab}
\]

2. Simplify:
\[
\frac{2 + \frac{1}{x}}{y}
\]

Answer:
\[
\left(\frac{2}{y}\right)\frac{2 + \frac{1}{x}}{y}
\]
\[
\frac{2x + 1}{y}
\]
\[
\frac{2x + 1}{y} \cdot \frac{y}{x}
\]
\[
\frac{2x^2 + y}{x}
\]

3. Simplify:
\[
\frac{1 - \frac{3}{x}}{2}
\]

Answer:
\[
\left(\frac{2}{x}\right)\frac{1 - \frac{3}{x}}{2}
\]
\[
\frac{x - 3}{2x}
\]
\[
\frac{x - 3}{2x}
\]
\[
\frac{x-3}{2} = \frac{x}{x} - \frac{3z}{2x}
\]
1.7 Example 4: Numerator/Denominator Fractions

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 6. Complex Fractions in the MAT096 Review website, and watch the streaming video for 4) Numerator/Denominator Fractions. Check your work with the answers on the following page.

1. Simplify:
   \[
   \frac{\frac{2}{a} + \frac{1}{b}}{\frac{1}{a^2} - \frac{1}{b}}
   \]

2. Simplify:
   \[
   \frac{\frac{2}{x} + \frac{1}{y}}{\frac{1}{x^2} + \frac{2}{y}}
   \]

3. Simplify:
   \[
   \frac{\frac{1}{3} - \frac{2}{5}}{\frac{2}{3} + \frac{1}{4}}
   \]
1.8 Answers to Example 4: Numerator/Denominator Fractions

1. Simplify:
\[ \frac{\frac{3}{2} + \frac{1}{b}}{\frac{1}{a^2} - \frac{2}{b}} \]

Answer:

\[ \frac{(\frac{3}{2}) \frac{a}{b} + \frac{1}{b} (\frac{1}{a^2})}{\frac{1}{a^2} - \frac{2}{b} (\frac{a}{b})} \]

\[ \frac{2b + a}{a^2 b} \cdot \frac{a^2 b}{2ab + a^2} \]

\[ \frac{2b + a}{ab} \cdot \frac{a^2 b}{b - 2a^2} \]

2. Simplify:
\[ \frac{\frac{2}{x} + \frac{1}{y}}{\frac{1}{x} + \frac{2}{y}} \]

Answer:

\[ \frac{(\frac{2}{x}) \frac{1}{x} + \frac{1}{y} (\frac{1}{y})}{\frac{1}{x} + \frac{2}{y} (\frac{1}{y})} \]

\[ \frac{2y + x}{xy} \cdot \frac{xy}{y + 2x} \]

\[ \frac{2y + x}{y + 2x} \]

3. Simplify:
\[ \frac{\frac{1}{x} - \frac{2}{y}}{\frac{1}{x} + \frac{1}{y}} \]

Answer:
\begin{align*}
\frac{5}{2} \cdot -\frac{2}{3} &= \frac{10}{3} \\
\frac{5}{2} \div \frac{3}{4} &= \frac{10}{3} \\
\frac{15}{4} - \frac{5}{3} &= \frac{12}{11} \\
\frac{1}{11} \cdot \frac{1}{11} &= \frac{1}{121}
\end{align*}
1.9   Example 5: Denominator Fractions

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 6. Complex Fractions in the MAT096 Review website, and watch the streaming video for 5) Denominator Fractions. Check your work with the answers on the following page.

1. Simplify:
   \[\frac{\frac{1}{a^2}}{\frac{1}{a} - \frac{1}{b}}\]

2. Simplify:
   \[\frac{\frac{1}{x^2}}{\frac{1}{x} + \frac{1}{y}}\]

3. Simplify:
   \[\frac{\frac{1}{x^2}}{\frac{1}{x} + \frac{1}{y}}\]
1.10 Answers to Example 5: Denominator Fractions

1. Simplify:
\[ \frac{\frac{1}{a}}{\frac{1}{b}} \]

Answer:
\[ \frac{1}{a} \cdot \frac{b}{b} - \frac{1}{b} \cdot \frac{a}{a} \]
\[ \frac{b^2}{a^2} \cdot \frac{1}{b-a} \]
\[ \frac{ab - a^2}{ab} \]

2. Simplify:
\[ \frac{\frac{1}{x^2}}{\frac{1}{x} + \frac{1}{y}} \]

Answer:
\[ \frac{1}{x^2} \cdot \frac{1}{\frac{y}{x} + \frac{x}{y}} \]
\[ \frac{1}{x^2} \cdot \frac{xy}{y + x} \]
\[ \frac{xy + x^2}{xy} \]

3. Simplify:
\[ \frac{\frac{1}{2}}{\frac{1}{5} + \frac{1}{5}} \]

Answer:
\[ \frac{\frac{1}{2}}{\frac{1}{5} \cdot \frac{1}{2} + \frac{1}{5} \cdot \frac{1}{2}} \]
\[ \frac{\frac{1}{5}}{\frac{1}{10}} \]
\[ \frac{\frac{2}{5}}{1} \]
2 Integer Exponents

2.1 Example 1: Numerical Examples

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 7. Integers Exponents in the MAT096 Review website, and watch the streaming video for 1) Numerical Examples. Check your work with the answers on the following page.

1. Simplify: $9^{-2}$

2. Simplify: $5^{-2}$

3. Simplify: $3^{-4}$
2.2 Answers to Example 1: Numerical Examples

1. Simplify: \(9^{-2}\)
   
   Answer:
   
   \[
   9^{-2} = \frac{1}{81}
   \]

2. Simplify: \(5^{-2}\)
   
   Answer:
   
   \[
   5^{-2} = \frac{1}{25}
   \]

3. Simplify: \(3^{-4}\)
   
   Answer:
   
   \[
   3^{-4} = \frac{1}{81}
   \]
2.3 Example 2: Variables Examples

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 7. Integers Exponents in the MAT096 Review website, and watch the streaming video for 2) Variable Examples. Check your work with the answers on the following page.

1. Simplify: \[
\frac{r^{-1}t^{-1}}{s^{-2}}
\]

2. Simplify: \[
\frac{m^{-2}n^{-2}}{p^{-1}}
\]

3. Simplify: \[
\frac{x^{-2}y^{3}}{z^{-2}}
\]
2.4 Answers to Example 2: Variable Examples

1. Simplify: \( \frac{t^{-1}r^{-1}}{s^{-2}} \)

Answer:

\( \frac{s^2}{rt} \)

2. Simplify: \( \frac{m^{-2}n^{-2}}{p^{-3}} \)

Answer:

\( \frac{p^3}{m^2n^2} \)

3. Simplify: \( \frac{y^3}{z^{-2}} \)

Answer:

\( \frac{z^2y^3}{x^2} \)
2.5 Example 3: Variables Examples with Reduction

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 7. Integers Exponents in the MAT096 Review website, and watch the streaming video for 3 Variables Examples with Reduction. Check your work with the answers on the following page.

1. Simplify:
   \[
   \frac{x^{-9}y^3}{x^{-4}y^{-7}}
   \]

2. Simplify:
   \[
   \frac{x^{-3}y^{-6}}{x^{-4}y^{-4}}
   \]

3. Simplify:
   \[
   \frac{x^{3}xy^{-3}x^{-5}}{x^{-4}z^{4}y}
   \]
2.6 Answers to Example 3: Variable Examples with Reduction

1. Simplify: \( \frac{x - 6y^3}{x - 4y - 7} \)

Answer:

\[ \frac{x^4y^7y^3}{y^{10}} \]

2. Simplify: \( \frac{x - 4y - 3}{x - 2y - 4} \)

Answer:

\[ \frac{x^2y^4}{x^4y^3} \]

3. Simplify: \( \frac{z^3x^4y^3}{z^3x - 3, x - 5} \)

Answer:

\[ \frac{z^3x^4y^3}{x^4z^3y^3y} \]

1 \[ \frac{z^3x^4}{x^4z^3y^3y} \]
2.7 Example 4: Raising a Rational Expression to a Power

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 7. Integers Exponents in the MAT096 Review website, and watch the streaming video for 4Raising a Rational Expression to a Power). Check your work with the answers on the following page.

1. Simplify: \((\frac{x^2y^{-4}}{z^3})^{-3}\)

2. Simplify: \((\frac{x^3y^{-3}}{z^4})^{-2}\)

3. Simplify: \((\frac{x^3y^4}{z^2})^{-1}\)
2.8 Answers to Example 4: Raising a Rational Expression to a Power

1. Simplify: 
\[(\frac{z^{-2}y^{-4}}{x^4})^{-3}\]

Answer:

\[
\frac{x^{12}y^{12}}{x^{12}y^{12}z^{6}}
\]

2. Simplify: 
\[(\frac{x^2y^{-3}}{z^3})^{-2}\]

Answer:

\[
\frac{x^{-4}y^6}{z^6}
\]

3. Simplify: 
\[(\frac{x^3y^3}{z^2})^{-1}\]

Answer:

\[
\frac{z^{-2}}{x^{3}y^3}
\]
3 Rational Exponents

3.1 Example 1: Unit Numerator Conversion Applications

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 8. Rational Exponents in the MAT096 Review website, and watch the streaming video for 1 Unit Numerator Conversion Applications). Check your work with the answers on the following page.

1. Convert to rational exponents:
   \[ \sqrt{z} \]

2. Convert to rational exponents:
   \[ \sqrt[3]{m} \]

3. Convert to rational exponents:
   \[ \sqrt[p]{p} \]
3.2 Answers to Example 1: Unit Numerator Conversion Applications

1. Simplify:
\[ \sqrt{z} \]
Answer:
\[ z^{\frac{1}{2}} \]

2. Simplify:
\[ \sqrt[3]{m} \]
Answer:
\[ m^{\frac{1}{3}} \]

3. Simplify:
\[ \sqrt[4]{p} \]
Answer:
\[ p^{\frac{1}{4}} \]
3.3 Example 2: Non-Unit Numerator and Denominator Applications

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 8. Rational Exponents in the MAT096 Review website, and watch the streaming video for 2 Non-Unit Numerator and Denominator Applications. Check your work with the answers on the following page.

1. Convert to rational exponents:
\[ \sqrt[3]{a^3} \]

2. Convert to rational exponents:
\[ \sqrt[3]{m^5} \]

3. Convert to rational exponents:
\[ \sqrt[4]{x^9} \]
3.4 Answers to Example 2: Non-Unit Numerator and Denominator Applications

1. Simplify: $\sqrt[3]{a^3}$

Answer:

$a^{\frac{3}{3}}$

2. Simplify: $\sqrt[5]{m^5}$

Answer:

$m^{\frac{5}{5}}$

3. Simplify: $\sqrt[9]{x^9}$

Answer:

$x^{\frac{9}{9}}$
3.5 Example 3: Distributive Law Applications

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 8. Rational Exponents in the MAT096 Review website, and watch the streaming video for 3 Distributive Law Applications). Check your work with the answers on the following page.

1. Convert to rational exponents and distribute:
\[ \frac{4}{\sqrt[5]{y^5}} \left( \sqrt[3]{y^3} + \sqrt[5]{y^2} \right) \]

2. Convert to rational exponents and distribute:
\[ \frac{3}{\sqrt[5]{x^5}} \left( \sqrt[5]{x^5} + \sqrt[5]{x^2} \right) \]

3. Convert to rational exponents and distribute:
\[ \frac{3}{\sqrt[2]{z^2}} \left( \sqrt[3]{x^3} - \sqrt[3]{z^3} \right) \]
3.6 Answers to Example 3: Distributive Law Applications

1. Simplify: $\sqrt[3]{y^5}(\sqrt[3]{y^3} + \sqrt[3]{y^2})$

Answer:

$$\frac{y^{\frac{5}{3}}(y^{\frac{3}{2}} + y^{\frac{2}{3}})}{y^{\frac{11}{3}} + y^{\frac{2}{3}}}$$

2. Simplify: $\sqrt[3]{x^5}(\sqrt[3]{x^5} + \sqrt[3]{x^2})$

Answer:

$$\frac{x^{\frac{5}{3}}(x^{\frac{4}{3}} + x^{\frac{2}{3}})}{x^{\frac{11}{3}} + x^{\frac{2}{3}}}$$

3. Simplify: $\sqrt[3]{z^2}(\sqrt[3]{z^3} - \sqrt[3]{z^3})$

Answer:

$$\frac{z^{\frac{3}{2}}(z^{\frac{3}{2}} - z^{\frac{3}{2}})}{z^{\frac{12}{6}} - z^{\frac{21}{6}}}$$
3.7 Example 4: Division Applications

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 8. Rational Exponents in the MAT096 Review website, and watch the streaming video for 4 Division Applications). Check your work with the answers on the following page.

1. Express your answer in simplest form with rational exponents:
\[
\frac{y^{\frac{4}{3}} \cdot x^{\frac{3}{8}}}{x^{\frac{1}{8}}}
\]

2. Express your answer in simplest form with rational exponents:
\[
\frac{\sqrt[3]{x^2}}{\sqrt[3]{x^4}}
\]

3. Express your answer in simplest form with rational exponents:
\[
\frac{\sqrt[3]{y^{2}}}{\sqrt[3]{y^{4}}}
\]
3.8 Answers to Example 4: Division Applications

1. Express your answer in simplest form with rational exponents:
\[ \frac{y^{\frac{4}{5}} \cdot x^{\frac{3}{8}}}{x^{\frac{5}{8}}} \]

Answer:
\[ y^{\frac{4}{5}} \cdot x^{\frac{3}{8} - \frac{5}{8}} = y^{\frac{4}{5}} \cdot x^{-\frac{1}{4}} \]

2. Express your answer in simplest form with rational exponents:
\[ \frac{\sqrt[3]{x^{\frac{4}{5}}}}{\sqrt[3]{x^{\frac{3}{5}}}} \]

Answer:
\[ x^{\frac{4}{3} - \frac{3}{5}} = x^{\frac{11}{15}} \]

3. Express your answer in simplest form with rational exponents:
\[ \frac{\sqrt[4]{y^{\frac{2}{5}}}}{\sqrt[5]{y^{\frac{3}{2}}}} \]

Answer:
\[ \frac{y^{\frac{2}{4} - \frac{3}{2}}}{y^{\frac{1}{12}}} = \frac{1}{y^{\frac{1}{12}}} \]
3.9 Example 5: Negative Exponent Applications

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 8. Rational Exponents in the MAT096 Review website, and watch the streaming video for 5 Negative Exponent Applications). Check your work with the answers on the following page.

1. Write without negative exponents:
\[
\frac{a^{\frac{1}{2}} b^{-\frac{1}{2}}}{a^{-\frac{1}{2}} b^{\frac{1}{2}}}
\]

2. Write without negative exponents:
\[
\frac{x^{-\frac{2}{3}} y^{\frac{2}{3}}}{x^{\frac{1}{3}} y^{\frac{2}{3}}}
\]

3. Write without negative exponents:
\[
\frac{a^{\frac{1}{2}} b^{-\frac{1}{3}}}{a^{-\frac{1}{2}} b^{\frac{1}{3}}}
\]
3.10 Answers to Example 5: Negative Exponent Applications

1. Write without negative exponents:
\[
\frac{a^{\frac{1}{3}}b^{\frac{1}{4}}}{a^{\frac{2}{3}}b^{\frac{3}{4}}}
\]
Answer:
\[
\frac{a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot b^{\frac{1}{4}}}{b^{\frac{3}{4}}}
\]
\[
\frac{a^{\frac{2}{3}} \cdot b^{\frac{1}{4}}}{b^{\frac{3}{4}}}
\]

2. Write without negative exponents:
\[
\frac{x^{\frac{2}{3}}y^{\frac{1}{2}}}{x^{\frac{1}{3}}y^{\frac{3}{4}}}
\]
Answer:
\[
\frac{1}{x^{\frac{2}{3}} \cdot x \cdot y^{\frac{1}{2}} \cdot y^{\frac{3}{4}}}
\]
\[
\frac{1}{x^{\frac{1}{3}} \cdot y^{\frac{3}{4}}}
\]

3. Write without negative exponents:
\[
\frac{a^{\frac{1}{2}}b^{\frac{1}{4}}}{a^{\frac{3}{4}}b^{\frac{3}{4}}}
\]
Answer:
\[
\frac{a^{\frac{1}{2}} \cdot a^{\frac{3}{4}} \cdot b^{\frac{1}{4}}}{b^{\frac{3}{4}}}
\]
\[
\frac{a^{\frac{5}{4}} \cdot b^{\frac{1}{4}}}{b^{\frac{3}{4}}}
\]
4 Rationalizing the Denominator

4.1 Example 1: Rationalizing Square Roots

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 9. Rationalizing the Denominator in the MAT096 Review website, and watch the streaming video for 1) Rationalizing Square Roots). Check your work with the answers on the following page.

1. Rationalize the denominator:
\[ \frac{1}{\sqrt{7}} \]

2. Rationalize the denominator:
\[ \frac{2}{\sqrt{8}} \]

3. Rationalize the denominator:
\[ \frac{2}{\sqrt{3}} \]
4.2 Answers to Example 1: Rationalizing Square Roots

1. Rationalize the denominator:
\[
\frac{1}{\sqrt{7}}
\]
Answer:
\[
\frac{1}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{1}{7}
\]

2. Rationalize the denominator:
\[
\frac{2}{\sqrt{5}}
\]
Answer:
\[
\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}
\]

3. Rationalize the denominator:
\[
\frac{2}{\sqrt{3}}
\]
Answer:
\[
\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}
\]
4.3 Example 2: Rationalizing Cube Roots

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 9. Rationalizing the Denominator in the MAT096 Review website, and watch the streaming video for 2) Rationalizing Cube Roots). Check your work with the answers on the following page.

1. Rationalize the denominator:
   \[
   \frac{3}{\sqrt[3]{1}}
   \]

2. Rationalize the denominator:
   \[
   \frac{2}{\sqrt[3]{9}}
   \]

3. Rationalize the denominator:
   \[
   \frac{7}{\sqrt[3]{8}}
   \]
4.4 Answers to Example 2: Rationalizing Cube Roots

1. Rationalize the denominator:
\[ \frac{3}{\sqrt[3]{2}} \]
Answer:
\[ \frac{3}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{64}}{\sqrt[3]{64}} \]
\[ = \frac{3}{\sqrt[3]{2}} \cdot \frac{4}{4} \]
\[ = \frac{3}{\sqrt[3]{2}} \cdot \frac{4}{4} \]
\[ = \frac{3}{\sqrt[3]{2}} \cdot \frac{4}{4} \]

2. Rationalize the denominator:
\[ \frac{2}{\sqrt[3]{3}} \]
Answer:
\[ \frac{2}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} \]
\[ = \frac{2}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} \]
\[ = \frac{2}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} \]

3. Rationalize the denominator:
\[ \frac{7}{\sqrt[3]{8}} \]
Answer:
\[ \frac{7}{\sqrt[3]{8}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} \]
\[ = \frac{7}{\sqrt[3]{8}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} \]
\[ = \frac{7}{\sqrt[3]{8}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} \]
4.5 Example 3: Distributing Radicals

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 9. Rationalizing the Denominator in the MAT096 Review website, and watch the streaming video for 3) Distributing Radicals). Check your work with the answers on the following page.

1. Distribute:
   
   \((4 - \sqrt{2})(4 + \sqrt{2})\)

2. Distribute:
   
   \((2 + \sqrt{3})(1 + \sqrt{2})\)

3. Distribute:
   
   \((3 + \sqrt{5})(3 - \sqrt{5})\)
4.6 Answers to Example 3: Distributing Radicals

1. Distribute:
   \((4 - \sqrt{2})(4 + \sqrt{2})\)
   Answer:

   \(16 - \sqrt{4} = 16 - 2 = 14\)

2. Distribute:
   \((2 + \sqrt{3})(1 + \sqrt{2})\)

   Answer:
   \(3 + 2\sqrt{2} + \sqrt{3} + \sqrt{6}\)

3. Distribute:
   \((3 + \sqrt{5})(3 - \sqrt{5})\)
   Answer:

   \(9 - \sqrt{25} = 9 - 5 = 4\)
4.7 Example 4: Rationalizing Denominators with Conjugates

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 9. Rationalizing the Denominator in the MAT096 Review website, and watch the streaming video for 4 Rationalizing Denominators with Conjugates). Check your work with the answers on the following page.

1. Rationalize the denominator:
   \[ \frac{-5}{1+\sqrt{5}} \]

2. Rationalize the denominator:
   \[ \frac{-2}{3-\sqrt{7}} \]

3. Rationalize the denominator:
   \[ \frac{4-\sqrt{2}}{1+\sqrt{2}} \]
4.8 Answers to Example 4: Rationalizing Denominators with Conjugates

1. Rationalize the denominator:
   \[
   \frac{-5}{1+\sqrt{5}}
   \]
   Answer:

   \[
   \frac{-5}{1+\sqrt{5}} \cdot \frac{1-\sqrt{5}}{1-\sqrt{5}} = \frac{-5(1-\sqrt{5})}{1-5} = \frac{-4}{-4} = 1
   \]

2. Rationalize the denominator:
   \[
   \frac{-2}{3-\sqrt{7}}
   \]
   Answer:

   \[
   \frac{-2}{3-\sqrt{7}} \cdot \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{-6-2\sqrt{7}}{9-7} = \frac{-6-2\sqrt{7}}{2}
   \]

3. Rationalize the denominator:
   \[
   \frac{4-\sqrt{2}}{1+\sqrt{2}}
   \]
   Answer:

   \[
   \frac{4-\sqrt{2}}{1+\sqrt{2}} \cdot \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{4\sqrt{2}-2-6\sqrt{2}}{1-2} = \frac{-2}{-1} = 2
   \]

   \[
   3\sqrt{2} - 6
   \]
4.9 Example 5: Adding then Rationalizing

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 9. Rationalizing the Denominator in the MAT096 Review website, and watch the streaming video for 5 Adding then Rationalizing). Check your work with the answers on the following page.

1. Add the real numbers, then rationalize the denominator:
\[ \frac{1}{\sqrt{2}} - \frac{4}{\sqrt{3}} \]

2. Add the real numbers, then rationalize the denominator:
\[ \frac{2}{\sqrt{5}} + \frac{3}{\sqrt{3}} \]

3. Add the real numbers, then rationalize the denominator:
\[ \frac{-2}{\sqrt{2}} - \frac{3}{\sqrt{10}} \]
4.10 Answers to Example 5: Adding then Rationalizing

1. Add the real numbers, then rationalize the denominator:
\[
\frac{\sqrt{3}}{\sqrt{6}} - 4\frac{\sqrt{2}}{\sqrt{6}}
\]
Answer:
\[
\frac{(\sqrt{3}) \cdot 1 - 4(\sqrt{2})}{\sqrt{3} - 4\sqrt{2}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{18} - 4\sqrt{12}}{3\sqrt{2} - 8\sqrt{3}}
\]

2. Add the real numbers, then rationalize the denominator:
\[
\frac{2}{\sqrt{3}} + \frac{3}{\sqrt{3}}
\]
Answer:
\[
\frac{(\sqrt{3}) \cdot 2 + 3(\sqrt{3})}{\sqrt{3}} = \frac{2\sqrt{15} + 3\sqrt{15} \cdot \sqrt{5}}{\sqrt{15}} = \frac{2\sqrt{75} + 3\sqrt{75}}{15} \cdot \frac{15}{15} = \frac{2\sqrt{5} + 5\sqrt{3}}{5}
\]

3. Add the real numbers, then rationalize the denominator:
\[
\frac{-2}{\sqrt{2}} - \frac{3}{\sqrt{10}}
\]
Answer:
\[
\frac{(\sqrt{10}) \cdot -2 - 3(\sqrt{2})}{\sqrt{10} - \sqrt{2} \cdot \sqrt{10}} = \frac{-2\sqrt{20} - 3\sqrt{2}}{\sqrt{20}} \cdot \frac{\sqrt{20}}{\sqrt{20}} = \frac{-2\sqrt{200} - 3\sqrt{20}}{20}
\]
\[
\frac{2\sqrt{200} - 3\sqrt{40}}{20} - \frac{20\sqrt{2} - 6\sqrt{10}}{20} - \frac{10\sqrt{2} - 3\sqrt{10}}{10}
\]
5 Division of Polynomials

5.1 Example 1: Distributing denominators

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 10. Division of Polynomials in the MAT096 Review website, and watch the streaming video for 1) Distributing denominators). Check your work with the answers on the following page.

1. Distribute the denominator:
\[
\frac{6x^2 + 3x - 6}{3x}
\]

2. Distribute the denominator:
\[
\frac{4x^2 - 8x + 6}{2x}
\]

3. Distribute the denominator:
\[
\frac{3x^3 - 12x^2 + 3x}{3x}
\]
5.2 Answers to Example 1: Distributing denominators

1. Distribute the denominator:
\[
\frac{6x^2 + 3x - 6}{3x}
\]

Answer:
\[
\frac{6x^2}{3x} + \frac{3x}{3x} - \frac{6}{3x} = 2x + 1 - \frac{2}{x}
\]

2. Distribute the denominator:
\[
\frac{4x^2 - 8x + 6}{2x}
\]

Answer:
\[
\frac{4x^2}{2x} - \frac{8x}{2x} + \frac{6}{2x} = 2x - 4 + \frac{3}{x}
\]

3. Distribute the denominator:
\[
\frac{3x^2 - 12x^2 + 3x}{3x}
\]

Answer:
\[
\frac{3x^3}{3x} - \frac{12x^2}{3x} + \frac{3x}{3x} = x^2 - 4x + 1
\]
5.3 Example 2: Division without remainder

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 10. Division of Polynomials in the MAT096 Review website, and watch the streaming video for 2) Division without remainder). Check your work with the answers on the following page.

1. Divide:
   \[(x^2 + 5x + 6) \div (x + 3)\]

2. Divide:
   \[(x^2 + 7x + 12) \div (x + 4)\]

3. Divide:
   \[(x^2 + 2x - 3) \div (x - 1)\]
5.4 Answers to Example 2: Division without remainder

1. Divide:
\((x^2 + 5x + 6) \div (x + 3)\)

Answer:

\[
\begin{array}{r}
\phantom{-}x + 2 \\
\hline
x + 3) x^2 + 5x + 6 \\
- (x^2 + 3x) \\
\hline
2x + 6 \\
- (2x + 6) \\
\hline
0
\end{array}
\]

2. Divide:
\((x^2 + 7x + 12) \div (x + 4)\)

Answer:

\[
\begin{array}{r}
\phantom{-}x + 3 \\
\hline
x + 4) x^2 + 7x + 12 \\
- (x^2 + 4x) \\
\hline
3x + 12 \\
- (3x + 12) \\
\hline
0
\end{array}
\]

3. Divide:
\((x^2 + 2x - 3) \div (x - 1)\)

Answer:

\[
\begin{array}{r}
\phantom{-}x + 3 \\
\hline
x - 1) x^2 + 2x - 3 \\
- (x^2 - x) \\
\hline
3x - 3
\end{array}
\]
\((3x - 3)\) \\
\[\] \\
\(0\)
5.5 Example 3: Division with remainder

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 10. Division of Polynomials in the MAT096 Review website, and watch the streaming video for 3) Division with remainder). Check your work with the answers on the following page.

1. Divide:
   \((x^2 + 5x - 13) \div (x + 3)\)

2. Divide:
   \((x^2 - 5x + 11) \div (x + 2)\)

3. Divide:
   \((2x^2 + 5x - 3) \div (x - 5)\)
5.6 Answers to Example 3: Division with remainder

1. Divide:
   \((x^2 + 5x - 13) \div (x + 3)\)

   Answer:
   \[
   \begin{array}{c}
   x + 2 - \frac{19}{x+3} \\
   x + 3) x^2 + 5x - 13 \\
   - (x^2 + 3x) \\
   \hline
   2x - 13 \\
   - (2x + 6) \\
   \hline
   -19
   \end{array}
   \]

2. Divide:
   \((x^2 - 5x + 11) \div (x + 2)\)

   Answer:
   \[
   \begin{array}{c}
   x - 7 - \frac{25}{x+2} \\
   x + 2) x^2 - 5x + 11 \\
   - (x^2 + 2x) \\
   \hline
   -7x + 11 \\
   -(-7x - 14) \\
   \hline
   -25
   \end{array}
   \]

3. Divide:
   \((2x^2 + 5x - 3) \div (x - 5)\)

   Answer:
   \[
   \begin{array}{c}
   2x + 15 + \frac{72}{x-5} \\
   x - 5) 2x^2 + 5x - 3 \\
   - (2x^2 - 10x) \\
   \hline
   15x - 3 \\
   - (15x - 75) \\
   \hline
   72
   \end{array}
   \]
5.7 Example 4: Division with missing terms without remainder

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 10. Division of Polynomials in the MAT096 Review website, and watch the streaming video for 4) Division with missing terms without remainder). Check your work with the answers on the following page.

1. Divide: 
   
   $(4x^3 + 2x^2 + 90) \div (x - 3)$

2. Divide: 
   
   $(3x^3 + 5x + 8) \div (x - 1)$

3. Divide: 
   
   $(2x^2 - 18) \div (x + 3)$
5.8 Answers to Example 4: Division with missing terms without remainder

1. Divide:
\[(4x^3 + 2x^2 + 90) \div (x - 3)\]

Answer:

\[
x + 3)4x^3 + 2x^2 + 0 \cdot x + 90 \quad \rightarrow \quad 4x^2 - 10x + 30 \]
\[-(4x^3 + 12x^2) \rightarrow -10x^2 + 0 \cdot x + 90 \]
\[-(30x + 90) \rightarrow 0 \]

2. Divide:
\[(3x^3 + 5x + 8) \div (x - 1)\]

Answer:

\[
x - 1)3x^3 + 0 \cdot x^2 + 5x - 8 \quad \rightarrow \quad 3x^2 + 3x + 8 \]
\[-(3x^3 - 3x^2) \rightarrow 3x^2 + 5x - 8 \]
\[-(3x - 3x) \rightarrow 8x - 8 \]
\[-(8x - 8) \rightarrow 0 \]

3. Divide:
\[(2x^2 - 18) \div (x + 3)\]

Answer:
\[
\begin{align*}
\frac{2x - 6}{x + 3} & \div 2x^2 + 0 \cdot x - 18 \\
\phantom{\frac{2x - 6}{x + 3}} & -(2x^2 + 6x) \\
\phantom{\frac{2x - 6}{x + 3}} & -6x - 18 \\
\phantom{\frac{2x - 6}{x + 3}} & -(-6x - 18) \\
\phantom{\frac{2x - 6}{x + 3}} & 0
\end{align*}
\]
5.9 Example 5: Division with missing terms and remainder

Directions: Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on 10. Division of Polynomials in the MAT096 Review website, and watch the streaming video for 5) Division with missing terms and remainder). Check your work with the answers on the following page.

1. Divide:
   \[(6x^3 + 8x + 14) \div (x + 2)\]

2. Divide:
   \[(2x^2 + 5) \div (x + 7)\]

3. Divide:
   \[(2x^3 - 3x + 1) \div (x - 5)\]
5.10 Answers to Example 5: Division with missing terms and remainder

1. Divide:
\[(6x^3 + 8x + 14) \div (x + 2)\]

Answer:

\[
\begin{array}{r}
\phantom{6x^2 - 12x + 32} - \frac{50}{x+2} \\
6x^2 - 12x + 32 - \frac{50}{x+2} \\
x + 2 | 6x^3 + 0 \cdot x^2 + 8x + 14 \\
-\frac{6x^3 + 12x^2}{-12x^2 + 8x + 14} \\
-\frac{-12x^2 - 24x}{32x + 14} \\
-\frac{-32x - 64}{-50}
\end{array}
\]

2. Divide:
\[(2x^2 + 5) \div (x + 7)\]

Answer:

\[
\begin{array}{r}
\phantom{2x - 14} + \frac{103}{x+7} \\
2x - 14 + \frac{103}{x+7} \\
x + 7 | 2x^2 + 0 \cdot x + 5 \\
-\frac{2x^2 + 14x}{-14x + 5} \\
-\frac{-14x - 98}{103}
\end{array}
\]

3. Divide:
\[(2x^3 - 3x + 1) \div (x - 5)\]

Answer:

\[
\begin{array}{r}
\phantom{2x^2 + 10x} + \frac{236}{x-5} \\
2x^2 + 10x + 47 + \frac{236}{x-5} \\
x - 5 | 2x^3 + 0 \cdot x^2 - 3x + 1
\end{array}
\]
\[
\begin{align*}
\frac{-(2x^3 - 10x^2)}{10x^2 - 3x + 1} & \quad \frac{-(10x^2 - 50x)}{47x + 1} \\
& \quad \frac{-(47x - 235)}{236}
\end{align*}
\]