# MAT096 Review Workbook
# Part III

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April 5, 2007

# 1 Factoring Trinomial

## 1.1 Example 1: Factoring Monomials

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *11. Factoring Trinomial* in the MAT096 Review website, and watch the streaming video for *1) Factoring Monomials.* Check your work with the answers on the following page.

1. Factor:
   $16x^2 + 4x - 12$

2. Factor:
   $3x^3 - 5x^2 + 15x$

3. Factor:
   $2a^2b^2 + 4ab^2 + 6ab$

## 1.2 Answers to Example 1: Factoring Monomials

1. Factor:
   $16x^2 + 4x - 12$

   **Answer:**

   $3x(x^2 - 2x + 5)$

2. Factor:
   $3x^3 - 5x^2 + 15x$

   **Answer:**

   $3x(x^2 - 2x + 5)$

3. Factor:
   $2a^2b^2 + 4ab^2 + 6ab$
   **Answer:**

   $2ab(ab + 2b + 3)$

## 1.3 Example 2A: Factoring Trinomials of the form ++/-+

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *11. Factoring Trinomials* in the MAT096 Review website, and watch the streaming video for *2A) Factoring Trinomials of the form ++/+-.* Check your work with the answers on the following page.

1. Factor:
   $x^2 + 5x + 6$

2. Factor:
   $x^2 - 5x + 6$

3. Factor:
   $x^2 + 7x + 12$

### 1.4 Answers to Example 2A: Factoring Trinomials of the form ++/-+

1. Factor:
   $x^2 + 5x + 6$

   **Answer:**

   $(x + 2)(x + 3)$

2. Factor:
   $x^2 - 5x + 6$

   **Answer:**

   $(x - 2)(x - 3)$

3. Factor:
   $x^2 + 7x + 12$
   **Answer:**

   $(x + 3)(x + 4)$

## 1.5 Example 2B: Factoring Trinomials of the form –/+-

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *11. Factoring Trinomials* in the MAT096 Review website, and watch the streaming video for *2B) Factoring Trinomials of the form –/+-*. Check your work with the answers on the following page.

1. Factor:
   $x^2 + 2x - 15$

2. Factor:
   $x^2 - x - 12$

3. Factor:
   $x^2 + 5x - 14$

## 1.6 Answers to Example 2B: Factoring Trinomials of the form −/+-

1. Factor:
   $x^2 + 2x - 15$

   **Answer:**

   $(x - 3)(x + 5)$

2. Factor:
   $x^2 - x - 12$

   **Answer:**

   $(x + 3)(x - 4)$

3. Factor:
   $x^2 + 5x - 14$
   **Answer:**

   $(x - 2)(x + 7)$

## 1.7 Example 3: Factoring the difference of two squares

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *11. Factoring Trinomials* in the MAT096 Review website, and watch the streaming video for *3) Factoring the difference of two squares.* Check your work with the answers on the following page.

1. Factor:
   $4a^2 - 81$

2. Factor:
   $16x^2 - 25$

3. Factor:
   $9x^2 - 49$

## 1.8 Answers to Example 3: Factoring the difference of two squares

1. Factor:
   $4a^2 - 81$

   **Answer:**

   $(2a - 9)(2a + 9)$

2. Factor:
   $16x^2 - 25$

   **Answer:**

   $(4x - 5)(4x + 5)$

3. Factor:
   $9x^2 - 49$
   **Answer:**

   $(3x - 7)(3x + 7)$

## 1.9 Example 4: Factoring perfect square trinomials

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *11. Factoring Trinomials* in the MAT096 Review website, and watch the streaming video for *4) Factoring perfect square trinomials*. Check your work with the answers on the following page.

1. Factor:
   $16x^2 - 14x + 9$

2. Factor:
   $9x^2 + 12x + 4$

3. Factor:
   $25x^2 - 40x + 16$

## 1.10 Answers to Example 4: Factoring perfect square trinomials

1. Factor:
   $16x^2 - 14x + 9$

   **Answer:**

   $(4x - 3)(4x - 3)$

2. Factor:
   $9x^2 + 12x + 4$

   **Answer:**

   $(3x + 2)(3x + 2)$

3. Factor:
   $25x^2 - 40x + 16$
   **Answer:**

   $(5x - 4)(5x - 4)$

# 2 Division of Rational Expressions

## 2.1 Example 1: Division of fractions

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *12. Division of Rational Expressions* in the MAT096 Review website, and watch the streaming video for *1) Division of fractions*. Check your work with the answers on the following page.

1. Divide:
$\frac{42}{35} \div \frac{18}{7}$

2. Divide:
$\frac{9}{5} \div \frac{3}{4}$

3. Divide:
$\frac{14}{54} \div \frac{4}{9}$

## 2.2 Answers to Example 1: Division of fractions

1. Divide:
   $\frac{42}{35} \div \frac{18}{7}$

   **Answer:**
   $\frac{42}{35} \times \frac{7}{18}$
   $\frac{7}{5} \times \frac{1}{3}$
   $\frac{7}{10}$

2. Divide:
   $\frac{9}{5} \div \frac{3}{4}$

   **Answer:**

   $\frac{9}{5} \times \frac{4}{3}$
   $\frac{3}{5} \times \frac{4}{1}$
   $\frac{12}{5}$

3. Divide:
   $\frac{14}{54} \div \frac{4}{9}$
   **Answer:**

   $\frac{14}{54} \times \frac{9}{4}$
   $\frac{7}{6} \times \frac{1}{2}$
   $\frac{7}{12}$

## 2.3 Example 2: Division of Rational Expressions without factoring

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *12. Division of Rational Expressions* in the MAT096 Review website, and watch the streaming video for *2) Division of Rational Expressions without factoring.* Check your work with the answers on the following page.

1. Divide:
   $\frac{r^2w^2}{x^3y^2} \div \frac{rw}{x^4y^5}$

2. Divide:
   $\frac{r^2s^2}{t^2} \div \frac{r^3s^4}{t^4}$

3. Divide:
   $\frac{x^3y^4}{a^2b^3} \div \frac{xy^2}{ab^2}$

## 2.4 Answers to Example 2: Division of Rational Expressions without factoring

1. Divide:
   $\frac{r^2w^2}{x^3y^2} \div \frac{rw}{x^4y^5}$

   **Answer:**
   $\frac{r^2w^2}{x^3y^2} \times \frac{x^4y^5}{rw}$
   $rxw^3$

2. Divide:
   $\frac{r^2s^2}{t^2} \div \frac{r^3s^4}{t^4}$
   **Answer:**

   $\frac{r^2s^2}{t^2} \times \frac{t^4}{r^3s^4}$
   $\frac{t^2}{rs^2}$

3. Divide:
   $\frac{x^3y^4}{a^2b^3} \div \frac{xy^2}{ab^2}$
   **Answer:**

   $\frac{x^3y^4}{a^2b^3} \times \frac{ab^2}{xy^2}$
   $\frac{x^2y^2}{ab}$

## 2.5 Example 3: Division of Rational Expressions with factoring

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *12. Division of Rational Expressions* in the MAT096 Review website, and watch the streaming video for *3) Division of Rational Expressions with factoring*. Check your work with the answers on the following page.

1. Divide:
   $\frac{x-2}{x+3} \div \frac{x^2-4}{x+3}$

2. Divide:
   x$\frac{x^2-9}{x+2} \div \frac{x-3}{x^2-4}$

3. Divide:
   $\frac{x^2+4x+4}{x^2-4} \div \frac{x+2}{x-2}$

## 2.6 Answers to Example 3: Division of Rational Expressions with factoring

1. Divide:
   $\frac{x-2}{x+3} \div \frac{x^2-4}{x+3}$

   **Answer:**
   $\frac{x-2}{x+3} \times \frac{x+3}{x^2-4}$
   $\frac{x-2}{x+3} \times \frac{x+3}{(x+2)(x-2)}$
   $\frac{1}{(x+2)}$

2. Divide:
   $\frac{x^2-9}{x+2} \div \frac{x-3}{x^2-4}$

   **Answer:**
   $\frac{(x+3)(x-3)}{x+2} \times \frac{(x+2)(x-2)}{x-3}$
   $(x+3)(x-2)$

3. Divide:
   $\frac{x^2+4x+4}{x^2-4} \div \frac{x+2}{x-2}$

   **Answer:**

   $\frac{(x+2)(x+2)}{(x-2)(x+2)} \times \frac{x-2}{x+2}$
   $\frac{1}{(x-2)} \times x-2$
   $\frac{x-2}{(x-2)}$
   $1$

## 2.7 Example 4: Division of Rational Expressions using factoring by grouping

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *12. Division of Rational Expressions* in the MAT096 Review website, and watch the streaming video for *4) Division of Rational Expressions using factoring by grouping* . Check your work with the answers on the following page.

1. Divide:
   $\frac{3x^2-2x-1}{x+1} \div \frac{x-1}{x+1}$

2. Divide:
   $\frac{2x^2-x-1}{x+2} \div \frac{x-1}{x+2}$

3. Divide:
   $\frac{3x^2+5x+2}{x-2} \div \frac{3x+2}{x^2-4}$

## 2.8 Answers to Example 3: Division of Rational Expressions using factoring by grouping

1. Divide:
   $\frac{3x^2-2x-1}{x+1} \div \frac{x-1}{x+1}$

   **Answer:**
   $\frac{3x^2-2x-1}{x+1} \times \frac{x+1}{x-1}$
   $\frac{(3x^2-3x)+(x-1)}{x+1} \times \frac{x+1}{x-1}$
   $\frac{3x(x-1)+1\cdot(x-1)}{x+1} \times \frac{x+1}{x-1}$
   $\frac{(x-1)(3x+1))}{x+1} \times \frac{x+1}{x-1}$
   $3x+1$

2. Divide:
   $\frac{2x^2-x-1}{x+2} \div \frac{x-1}{x+2}$

   **Answer:**
   $\frac{2x^2-x-1}{x+2} \times \frac{x+2}{x-1}$
   $\frac{(2x^2-2x)+(x-1)}{x+2} \times \frac{x+2}{x-1}$
   $\frac{2x(x-1)+1\cdot(x-1)}{x+2} \times \frac{x+2}{x-1}$
   $\frac{(x-1)(2x+1))}{x+2} \times \frac{x+2}{x-1}$
   $(2x+1)$

3. Divide:
   $\frac{3x^2+5x+2}{x-2} \div \frac{3x+2}{x^2-4}$

   **Answer:**

   $\frac{3x^2+5x+2}{x-2} \times \frac{3x+2}{x^2-4}$
   $\frac{(3x^2+3x)+(2x+2)}{x-2} \times \frac{(x+2)(x-2)}{3x+2}$
   $\frac{3x(x+1)+2(x+1)}{x-2} \times \frac{(x+2)(x-2)}{3x+2}$

$\frac{(x+1)(3x+2))}{x-2} \times \frac{(x+2)(x-2)}{3x+2}$
$(x+1)(x+2)$

# 3 Equations with Radicals

## 3.1 Example 1: Simple Equations

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *13. Equations with Radicals* in the MAT096 Review website, and watch the streaming video for *1) Simple Equations*. Check your work with the answers on the following page.

1. Solve for x:
   $\sqrt{4x} = 2$

2. Solve for x:
   $\frac{\sqrt{x}}{2} = 3$

3. Solve for x:
   $5\sqrt{y} = 10$

## 3.2 Answers to Example 1: Simple Equations

1. Solve for x:
   $\sqrt{4x} = 2$

   **Answer:**

   $(\sqrt{4x})^2 = (2)^2$
   $4x = 4$
   $\frac{4x}{4} = \frac{4}{4}$
   $x = 1$

2. Solve for x:
   $\frac{\sqrt{x}}{2} = 3$

   **Answer:**

   $(2)\frac{\sqrt{x}}{2} = 3(2)$
   $\sqrt{x} = 6$
   $(\sqrt{x})^2 = 6^2$
   $x = 36$

3. Solve for x:
   $5\sqrt{y} = 10$

   **Answer:**

   $5\sqrt{y} = 10$
   $\frac{5\sqrt{y}}{5} = \frac{10}{5}$
   $(\sqrt{y})^2 = 2^2$
   $y = 4$

## 3.3 Example 2: Equations with roots of binomials

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *13. Equations with Radicals* in the MAT096 Review website, and watch the streaming video for *2) Equations with roots of binomials.* Check your work with the answers on the following page.

1. Solve for x:
   $\sqrt{3x-2} - 5 = 2$

2. Solve for x:
   $\sqrt{2x+2} - 4 = 2$

3. Solve for x:
   $\sqrt{5x+1} + 7 = 11$

## 3.4 Answers to Example 2: Equations with roots of binomials

1. Solve for x:
   $\sqrt{3x-2}-5=2$

   **Answer:**

   $\sqrt{3x-2}-5=2$
   $\underline{\qquad\qquad +5=+5}$
   $(\sqrt{3x-2})^2=(7)^2$
   $3x-2=49$
   $\underline{\quad +2=+2}$
   $3x=51$
   $\frac{3x}{3}=\frac{51}{3}$
   $x=17$

2. Solve for x:
   $\sqrt{2x+2}-4=2$

   **Answer:**

   $\sqrt{2x+2}-4=2$
   $\underline{\qquad\qquad +4=+4}$
   $(\sqrt{2x+2})^2=(6)^2$
   $2x+2=36$
   $\underline{\quad -2=-2}$
   $2x=34$
   $\frac{2x}{2}=\frac{34}{2}$
   $x=17$

3. Solve for x:
   $\sqrt{5x+1}+7=11$

**Answer:**

$$\begin{array}{l}
\sqrt{5x+1}+7=11 \\
\underline{\qquad\qquad -7=-7} \\
(\sqrt{5x+1})^2=(4)^2 \\
5x+1=16 \\
\underline{\quad -1=-1} \\
\frac{5x}{5}=\frac{15}{5} \\
5x=3 \\
x=3
\end{array}$$

## 3.5 Example 3: Isolating the radical

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *13. Equations with Radicals* in the MAT096 Review website, and watch the streaming video for *3) Isolating the radical*. Check your work with the answers on the following page.

1. Solve for x:
   $x + \sqrt{2x + 10} - 5 = 0$

2. Solve for x:
   $-x + \sqrt{x + 1} + 2 = 10$

3. Solve for x:
   $4 + \sqrt{x - 2} + x = 5$

## 3.6 Answers to Example 2: Isolating the radical

1. Solve for x:
   $x + \sqrt{2x+10} - 5 = 0$

   **Answer:**

   $$\begin{array}{l} x + \sqrt{2x+10} - 5 = 0 \\ \underline{-x \qquad\qquad\qquad + 5 = +5 - x} \\ (\sqrt{2x+10})^2 = (5-x)^2 \\ 2x + 10 = 25 - 10x + x^2 \\ \underline{-2x - 10 = -2x - 10} \\ \quad 0 = 15 - 12x + x^2 \\ \underline{-15 = -15} \\ \quad -15 = -12x + x^2 \\ \underline{\qquad 36 = 36} \\ \quad 21 = 36 - 12x + x^2 \\ \quad \pm\sqrt{21} = \sqrt{(-6+x)^2} \\ \quad \pm\sqrt{21} = -6 + x \\ \underline{\qquad\quad 6 = 6} \\ \quad 6 \pm \sqrt{21} = x \end{array}$$

2. Solve for x:
   $-x + \sqrt{x+1} + 2 = 10$

   **Answer:**

   $$\begin{array}{l} -x + \sqrt{x+1} + 2 = 10 \\ \underline{+x \qquad\qquad\qquad - 2 = -2 + x} \\ \sqrt{x+1} = 10 + x - 2 \\ (\sqrt{x+1})^2 = (8+x)^2 \\ x + 1 = 64 + 16x + x^2 \\ \underline{-x - 1 = -x - 1} \\ \quad 0 = 65 + 15x + x^2 \\ \underline{-65 = -65} \end{array}$$

$$-65 = 15x + x^2$$
$$\frac{225}{4} = \frac{225}{4}$$
$$-65 + \frac{225}{4} = \frac{225}{4} + 15x + x^2$$
$$\pm\sqrt{\frac{-35}{4}} = \sqrt{(\frac{15}{4} + x)^2}$$
$$\pm\sqrt{\frac{-35}{4}} = \frac{15}{4} + x$$
$$\frac{-15}{4} = \frac{15}{4}$$
$$\frac{-15}{4} \pm \sqrt{\frac{-35}{4}} = x$$

3. Solve for x:
   $4 + \sqrt{x-2} + x = 5$

   **Answer:**

$$4 + \sqrt{x-2} + x = 5$$
$$-4 \qquad - x = -4 - x$$
$$\sqrt{x-2} = 1 - x$$
$$(\sqrt{x-2})^2 = (1-x)^2$$
$$x - 2 = 1 - 2x + x^2$$
$$-x + 2 = -x + 2$$
$$0 = 3 - 3x + x^2$$
$$-3 = -3$$
$$-3 = -3x + x^2$$
$$\frac{9}{4} = \frac{9}{4}$$
$$-3 + \frac{9}{4} = \frac{9}{4} - 3x + x^2$$
$$\pm\sqrt{\frac{-3}{4}} = \sqrt{(\frac{-3}{2} + x)^2}$$
$$\frac{3}{2} = \frac{3}{2}$$
$$\frac{3}{2} \pm \frac{\sqrt{-3}}{2} = \frac{-3}{2} + x$$
$$\frac{3}{2} \pm \frac{\sqrt{-3}}{4} = x$$

## 3.7 Example 4: Squaring twice

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *13. Equations with Radicals* in the MAT096 Review website, and watch the streaming video for *4) Squaring twice* . Check your work with the answers on the following page.

1. Solve for x:
   $\sqrt{x+5} + \sqrt{x} = 1$

2. Solve for x:
   $\sqrt{x+2} - \sqrt{x} = 3$

3. Solve for x:
   $\sqrt{3x+1} + \sqrt{x+1} = 4$

## 3.8 Answers to Example 4: Squaring twice

1. Solve for x:
   $\sqrt{x+5} + \sqrt{x} = 1$

   **Answer:**

   $\sqrt{x+5} + \sqrt{x} = 1$
   $\underline{\qquad -\sqrt{x} \qquad - \sqrt{x}}$
   $(\sqrt{x+5})^2 = (1 - \sqrt{x})^2$
   $x + 5 = 1 - 2\sqrt{x} + x$
   $\underline{-x - 1 = -x - 1}$
   $4 = -2\sqrt{x}$
   $\frac{4}{-2} = \frac{-2\sqrt{x}}{-2}$
   $(-2)^2 = (\sqrt{x})^2$
   $4 = x$

2. Solve for x:
   $\sqrt{x+2} - \sqrt{x} = 3$

   **Answer:**

   $\sqrt{x+2} - \sqrt{x} = 3$
   $\underline{\qquad \sqrt{x} \qquad \sqrt{x}}$
   $(\sqrt{x+2})^2 = (3 + \sqrt{x})^2$
   $x + 2 = 9 + 6\sqrt{x} + x$
   $\underline{-x - 9 = -x - 9}$
   $-7 = 6\sqrt{x}$
   $\frac{-7}{6} = \frac{6\sqrt{x}}{6}$
   $(\frac{-7}{6})^2 = (\sqrt{x})^2$
   $\frac{49}{36} = x$

3. Solve for x:
   $\sqrt{3x+1} + \sqrt{x+1} = 4$

**Answer:**

$\sqrt{3x+1} + \sqrt{x+1} = 4$

$-\sqrt{x+1} \qquad = -\sqrt{x+1}$

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$(\sqrt{3x+1})^2 = (4 - \sqrt{x+1})^2$

$3x + 1 = 16 - 8\sqrt{x+1} + (x+1)$

$-(x+1) - 16 = -(x+1) - 16$

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$\frac{2x-16}{-8} = \frac{-8\sqrt{x+1}}{-8}$

$(\frac{-1}{4}x + 2)^2 = (\sqrt{x+1})^2$

$(\frac{-1}{4}x + 2)^2 = (\sqrt{x+1})^2$

$\frac{x^2}{16} - 1 + 4 = x + 1$

$(16)\frac{x^2}{16} + 3 = (16)x + 1$

$16x^2 + 48 = 16x + 16$

$-16x - 16 = -16x - 16$

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$(\frac{1}{16})16x^2 - 16x + 32 = 0(\frac{1}{16})$

$x^2 - 2x + 2 = 0$

$-2 = -2$

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$x^2 - 2x = -2$

$+1 = +1$

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$x^2 - 2x + 1 = -1$

$\sqrt{(x+1)^2} = \sqrt{-1}$

$x + 1 = \pm\sqrt{-1}$

$-1 = -1$

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$x = \pm\sqrt{-1} - 1$

## 3.9 Example 5: Additional example

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *13. Equations with Radicals* in the MAT096 Review website, and watch the streaming video for *5) Additional example*. Check your work with the answers on the following page.

1. Solve for x:
   $2x + \sqrt{x+3} = 4$

2. Solve for x:
   $x - 2\sqrt{x-2} = 6$

3. Solve for x:
   $3x + \sqrt{x+5} = 10$

## 3.10 Answers to Example 5: Additional example

1. Solve for x:
   $2x + \sqrt{x+3} = 4$

   **Answer:**

   $2x + \sqrt{x+3} = 4$
   $\underline{\quad -2x \qquad -2x}$
   $(\sqrt{x+3})^2 = (4-2x)^2$
   $x + 3 = 16 - 16x + 4x^2$
   $\underline{-x - 3 = -x - 3}$
   $(0 = 13 - 15x + 4x^2)(\frac{1}{4})$
   $0 = \frac{13}{4} - \frac{15}{4}x + x^2$
   $\underline{\quad -\frac{13}{4} \qquad -\frac{13}{4}}$
   $-\frac{13}{4} = -\frac{15}{4}x + x^2$
   $\underline{\quad +\frac{225}{64} \qquad +\frac{225}{64}}$
   $\frac{47}{64} = \frac{225}{64} - \frac{15}{4}x + x^2$
   $\pm\sqrt{\frac{47}{64}} = \sqrt{(\frac{15}{8} + x)^2}$
   $\pm\sqrt{\frac{47}{64}} = \frac{15}{8} + x$
   $\underline{\quad -\frac{15}{8} \qquad -\frac{15}{8}}$
   $-\frac{15}{8} \pm \sqrt{\frac{47}{64}} = +x$

2. Solve for x:
   $x - 2\sqrt{x-2} = 6$

   **Answer:**

   $x - 2\sqrt{x-2} = 6$
   $\underline{-x \qquad\qquad -x}$
   $\frac{-2\sqrt{x-2}}{-2} = \frac{6-x}{-2}$
   $(\sqrt{x-2})^2 = (-3 + \frac{x}{2})^2$
   $(4)(x-2)) = (9 - 3x + \frac{x^2}{4})(4)$

$4x - 8 = 36 - 12x + x$
$4x - 8 = 36 - 11x$
$\underline{\quad +11x \qquad + 11x}$
$15x - 8 = 36$
$\underline{\quad +8 \qquad + 8}$
$\frac{15x}{15} = \frac{44}{15}$
$x = \frac{44}{15}$

3. Solve for x:
$3x + \sqrt{x+5} = 10$

**Answer:**

$3x + \sqrt{x+5} = 10$
$\underline{-3x \qquad\qquad - 3x}$
$(\sqrt{x+5})^2 = (10 - 3x)^2$
$x - 5 = 100 - 60x + 9x^2$
$\underline{-x + 5 = -x + 5}$
$(0 = 105 - 61x + 9x^2)(\frac{1}{9})$
$0 = \frac{105}{9} - \frac{61}{9}x + x^2$
$\underline{\quad -\frac{105}{9} \qquad - \frac{105}{9}}$
$-\frac{105}{9} = -\frac{61}{9}x + x^2$
$\underline{\quad +\frac{3721}{324} \qquad + \frac{3721}{324}}$
$\frac{-56}{324} = \frac{3721}{324} - \frac{61}{9}x + x^2$
$\pm\sqrt{\frac{-56}{324}} = \sqrt{(\frac{-61}{9} + x)^2}$
$\pm\sqrt{\frac{-56}{324}} = \frac{-61}{9} + x$
$\underline{\quad +\frac{61}{9} \qquad + \frac{61}{9}}$
$\frac{61}{9} \pm \frac{\sqrt{14}}{9} = x$

# 4 Completing the Square

## 4.1 Example 1: Completing the square with positive coefficients

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *14. Completing the Square* in the MAT096 Review website, and watch the streaming video for *1) Completing the square with negative constants.* Check your work with the answers on the following page.

1. Solve for x by completing the square:
   $x^2 + 14x + 2 = 0$

2. Solve for x by completing the square:
   $x^2 - 12x + 1 = 0$

3. Solve for x by completing the square:
   $x^2 + 8x + 3 = 0$

## 4.2 Answers to Example 1: Completing the Square with positive constants

1. Solve for x by completing the square:
   $x^2 + 14x + 2 = 0$

   **Answer:**

$$
\begin{array}{l}
x^2 + 14x + 2 = 0 \\
\qquad\quad -2 \qquad -2 \\
\hline
x^2 + 14x = -2 \\
\qquad\quad +49 \qquad +49 \\
\hline
x^2 + 14x + 49 = 47 \\
\sqrt{(x+7)^2} = \sqrt{47} \\
x + 7 = \pm\sqrt{47} \\
\qquad\quad -7 \qquad -7 \\
\hline
x = \pm\sqrt{47} - 7
\end{array}
$$

2. Solve for x by completing the square:
   $x^2 - 12x + 1 = 0$

   **Answer:**

$$
\begin{array}{l}
x^2 - 12x + 1 = 0 \\
\qquad\quad -1 \qquad -1 \\
\hline
x^2 - 12x = -1 \\
\qquad\quad +36 \qquad +36 \\
\hline
x^2 - 12x = 35 \\
\sqrt{(x-6)^2} = \sqrt{35} \\
x - 6 = \pm\sqrt{35} \\
\qquad\quad +6 \qquad +6 \\
\hline
x = \pm\sqrt{35} + 6
\end{array}
$$

3. Solve for x by completing the square:
   $x^2 + 8x + 3 = 0$

**Answer:**

$$
\begin{array}{l}
x^2 + 8x + 3 = 0 \\
\underline{\qquad\quad -3 \qquad\quad -3} \\
x^2 + 8x = -3 \\
\underline{\qquad\quad +16 \qquad\quad +16} \\
x^2 + 8x = 13 \\
\sqrt{(x+4)^2} = \sqrt{13} \\
x + 4 = \pm\sqrt{13} \\
\underline{\qquad\quad -4 \qquad\quad -4} \\
x = \pm\sqrt{13} - 4
\end{array}
$$

## 4.3 Example 2: Proof of the quadratic equation

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *14. Completing the Square* in the MAT096 Review website, and watch the streaming video for *2) Proof of the quadratic equation*. Check your work with the answers on the following page.

1. Repeat the proof of the quadratic equation for:
   $rx^2 + sx + t = 0$

2. Repeat the proof of the quadratic equation for:
   $wx^2 + vx + p = 0$

3. Repeat the proof of the quadratic equation for:
   $dx^2 + ex + f = 0$

## 4.4 Answers to Example 2: Proof of the quadratic equation

1. Repeat the proof of the quadratic equation for:
   $rx^2 + sx + t = 0$

   **Answer:**

   $rx^2 + sx + t = 0$
   $\underline{\quad -t \qquad - t}$
   $rx^2 + sx = -t$
   $(\frac{1}{r})(rx^2 + sx) = (-t)(\frac{1}{r})$
   $x^2 + \frac{s}{r}x = \frac{-t}{r}$
   $\underline{\quad +\frac{s^2}{4r^2} \qquad + \frac{s^2}{4r^2}}$
   $x^2 + \frac{s}{r}x = \frac{-t}{r} + \frac{s^2}{4r^2}$
   $\sqrt{(x + \frac{s}{2r})^2} = \sqrt{\frac{s^2-4rt}{4r^2}}$
   $|x + \frac{s}{2r}| = \frac{\sqrt{s^2-4rt}}{2r}$
   $x + \frac{s}{2r} = \pm\frac{\sqrt{s^2-4rt}}{2r}$
   $\underline{\quad -\frac{s}{2r} \qquad - \frac{s}{2r}}$
   $x = \frac{-s\pm\sqrt{s^2-4rt}}{2r}$

2. Repeat the proof of the quadratic equation for:
   $wx^2 + vx + p = 0$

   **Answer:**

   $wx^2 + vx + p = 0$
   $\underline{\quad -p \qquad - p}$
   $wx^2 + vx = -p$
   $(\frac{1}{w})(wx^2 + vx) = (-p)(\frac{1}{w})$
   $x^2 + \frac{v}{w}x = \frac{-p}{w}$
   $\underline{\quad +\frac{v^2}{4w^2} \qquad + \frac{v^2}{4w^2}}$

$x^2 + \frac{v}{w}x + \frac{v^2}{4w^2} = \frac{-p}{w} + \frac{v^2}{4w^2}$
$\sqrt{(x + \frac{v}{2w})^2} = \sqrt{\frac{v^2-4wp}{4w^2}}$
$|x + \frac{v}{2w}| = \frac{\sqrt{v^2-4wp}}{2w}$
$x + \frac{v}{2w} = \pm\frac{\sqrt{v^2-4wp}}{2w}$
$\underline{\quad -\frac{v}{2w} \qquad -\frac{v}{2w}}$
$x = \frac{-v\pm\sqrt{v^2-4wp}}{2w}$

3. Repeat the proof of the quadratic equation for:
   $dx^2 + ex + f = 0$

   **Answer:**

$dx^2 + ex + f = 0$
$\underline{\quad -f \qquad -f}$
$dx^2 + ex = -f$
$(\frac{1}{d})(dx^2 + ex) = (-f)(\frac{1}{d})$
$x^2 + \frac{e}{d}x = \frac{-f}{d}$
$\underline{\quad +\frac{e^2}{4d^2} \qquad +\frac{e^2}{4d^2}}$
$x^2 + \frac{e}{d}x + \frac{e^2}{4d^2} = \frac{-f}{d} + \frac{e^2}{4d^2}$
$\sqrt{(x + \frac{e}{2d})^2} = \sqrt{\frac{e^2-4df}{4d^2}}$
$|x + \frac{e}{2d}| = \frac{\sqrt{e^2-4df}}{2d}$
$x + \frac{e}{2d} = \pm\frac{\sqrt{e^2-4df}}{2d}$
$\underline{\quad -\frac{e}{2d} \qquad -\frac{e}{2d}}$
$x = \frac{-e\pm\sqrt{e^2-4df}}{2d}$

## 4.5 Example 3: Application of the quadratic equation

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *14. Completing the Square* in the MAT096 Review website, and watch the streaming video for *3) Application of the quadratic equation*. Check your work with the answers on the following page.

1. Solve the following using the quadratic equation:
   $2x^2 + 3x - 10 = 0$

2. Solve the following using the quadratic equation:
   $x^2 - 2x + 5 = 0$

3. Solve the following using the quadratic equation:
   $3x^2 - 4x + 6 = 0$

## 4.6 Answers to Example 3: Application of the quadratic equation

1. Solve the following using the quadratic equation:
   $2x^2 + 3x - 10 = 0$
   **Answer:**

   $2x^2 + 3x - 10 = 0$
   $a = 2,\ b = 3,\ c = -10$

   $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
   $x = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-10)}}{2(2)}$
   $x = \frac{-3 \pm \sqrt{9+80}}{4}$
   $x = \frac{-3 \pm \sqrt{89}}{4}$

2. Solve the following using the quadratic equation:
   $x^2 - 2x + 5 = 0$
   **Answer:**

   $x^2 - 2x + 5 = 0$
   $a = 1,\ b = -2,\ c = -5$

   $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
   $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$
   $x = \frac{4 \pm \sqrt{4-20}}{2}$
   $x = \frac{4 \pm \sqrt{-16}}{2}$
   $x = \frac{4 \pm 4i}{2}$
   $x = 2 \pm 2i$

3. Solve the following using the quadratic equation:
   $3x^2 - 4x + 6 = 0$

**Answer:**

$3x^2 - 4x + 6 = 0$
$a = 3,\ b = -4,\ c = 6$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(6)}}{2(3)}$
$x = \frac{4 \pm \sqrt{16 - 72}}{6}$
$x = \frac{2 \pm \sqrt{14}i}{3}$

## 4.7 Example 4: Imaginary solution

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *14. Completing the Square* in the MAT096 Review website, and watch the streaming video for *4) Imaginary solution*. Check your work with the answers on the following page.

1. Solve by completing the square:
   $x^2 + 2x + 25 = 0$

2. Solve the following using the quadratic equation:
   $x^2 - 2x + 5 = 0$

3. Solve the following using the quadratic equation:
   $3x^2 - 4x + 6 = 0$

## 4.8 Answers to Example 4: Imaginary solution

1. Solve by completing the square:
   $x^2 + 2x + 25 = 0$
   **Answer:**

   $$\begin{array}{l} x^2 + 2x + 25 = 0 \\ \underline{\qquad\quad -25 \qquad -25} \\ x^2 + 2x = -25 \\ \underline{\qquad\quad +1 \qquad +1} \\ \sqrt{(x+1)^2} = \sqrt{-24} \\ x + 1 = \pm\sqrt{-24} \\ \underline{\qquad\quad -1 \qquad -1} \\ x = -1 \pm \sqrt{-24} \\ x = -1 \pm 2\sqrt{6}i \end{array}$$

2. Solve by completing the square:
   $x^2 - 4x + 16 = 0$
   **Answer:**

   $$\begin{array}{l} x^2 - 4x + 16 = 0 \\ \underline{\qquad\quad -16 \qquad -16} \\ x^2 - 4x = -16 \\ \underline{\qquad\quad +4 \qquad +4} \\ \sqrt{(x-2)^2} = \sqrt{-12} \\ x - 2 = \pm\sqrt{-12} \\ \underline{\qquad\quad +2 \qquad +2} \\ x = 2 \pm \sqrt{12}i \end{array}$$

3. Solve by completing the square:
   $x^2 + 6x + 40 = 0$
   **Answer:**

$$
\begin{array}{l}
x^2 + 6x + 40 = 0 \\
\underline{\qquad\quad -40 \qquad\quad -40} \\
x^2 + 6x = -40 \\
\underline{\qquad\quad +9 \qquad\quad +9} \\
\sqrt{(x+3)^2} = \sqrt{-31} \\
x + 3 = \pm\sqrt{-31} \\
\underline{\qquad\quad -3 \qquad\quad -3} \\
x = -3 \pm \sqrt{31}i
\end{array}
$$

# 5 Applications of quadratic equations

## 5.1 Example 1: Area Application I

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *15. Applications of quadratic equations* in the MAT096 Review website, and watch the streaming video for *1) Area Application I*. Check your work with the answers on the following page.

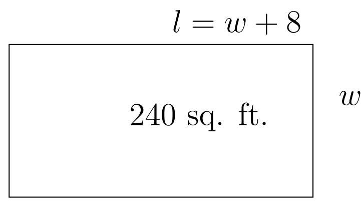
1. A rectangle is 8 ft. longer than it is wide. It's area is 240 square feet. Find its width.

2. A rectangle is 6 ft. longer than it is wide. It's area is 160 square feet. Find its width.

3. A rectangle is 2 ft. longer than it is wide. It's area is 143 square feet. Find its width.

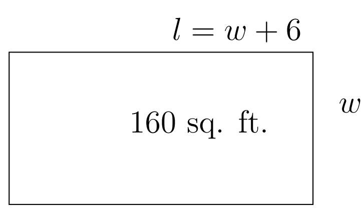
## 5.2 Answers to Example 1: Area Application I

1. A rectangle is 8 ft. longer than it is wide. It's area is 240 square feet. Find its width.
   **Answer:**




$w(w+8) = 240$
$w^2 + 8w = 240$
$\underline{\qquad +16 \qquad + 16}$
$w^2 + 8w + 16 = 256$
$\sqrt{(w+4)^2} = \sqrt{256}$
$w + 4 = \pm 16$
$\underline{\qquad -4 \qquad - 4}$
$w = -4 \pm 16$
$w_a = 12$
$w_b = -20$ extraneous root

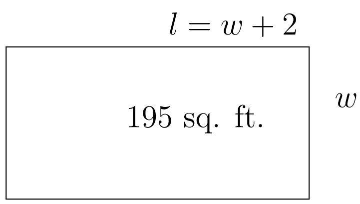
2. A rectangle is 6 ft. longer than it is wide. It's area is 160 square feet. Find its width.
   **Answer:**




$w(w+6) = 160$
$w^2 + 6w = 160$
$\underline{\qquad +9 \qquad + 9}$
$w^2 + 6w + 9 = 169$
$\sqrt{(w+3)^2} = \sqrt{169}$
$w + 3 = \pm 13$

$\underline{\quad -3 \qquad -3}$
$w = -3 \pm 13$
$w_a = 10$
$w_b = -16$ extraneous root

3. A rectangle is 2 ft. longer than it is wide. It's area is 195 square feet. Find its width.
**Answer:**




$w(w + 2) = 195$
$w^2 + 2w = 195$
$\underline{\quad +1 \qquad +1}$
$w^2 + 2w + 1 = 196$
$\sqrt{(w + 1)^2} = \sqrt{196}$
$w + 1 = \pm 14$
$\underline{\quad -1 \qquad -1}$
$w = -1 \pm 14$
$w_a = 13$
$w_b = -15$ extraneous root

## 5.3 Example 2: Area Application 2

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *15. Applications of quadratic equations* in the MAT096 Review website, and watch the streaming video for *1) Area Application 2*. Check your work with the answers on the following page.

1. The vertical edge of a triangular sail must be 7 feet longer than the foot of the sail. What is the length of the vertical edge if the area is 89 square feet?

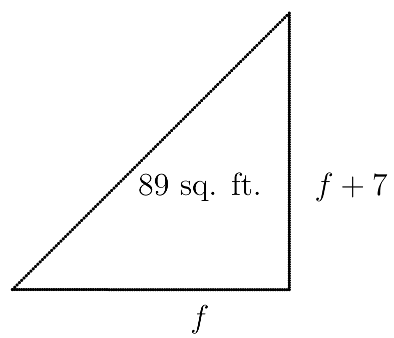
2. The vertical edge of a triangular sail must be 9 feet longer than the foot of the sail. What is the length of the vertical edge if the area is 95 square feet?

3. The vertical edge of a triangular sail must be 5 feet longer than the foot of the sail. What is the length of the vertical edge if the area is 102 square feet?

## 5.4 Answers to Example 2: Area Application 2

1. The vertical edge of a triangular sail must be 7 feet longer than the foot of the sail. What is the length of the vertical edge if the area is 99 square feet?

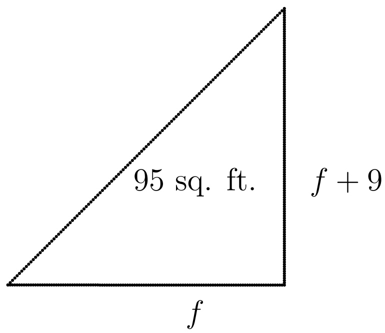
   **Answer:**




$99 = (\frac{1}{2})f(f + 7)$
$(2)99 = (2)(\frac{1}{2})f(f + 7)$
$198 = f^2 + 7f$
$\underline{\quad -198 \qquad - 198}$
$0 = f^2 + 7f - 198$
$0 = (f - 11)(f + 18)$
$f_a = 11$
$f_b = -18$ extraneous root

2. The vertical edge of a triangular sail must be 9 feet longer than the foot of the sail. What is the length of the vertical edge if the area is 95 square feet?

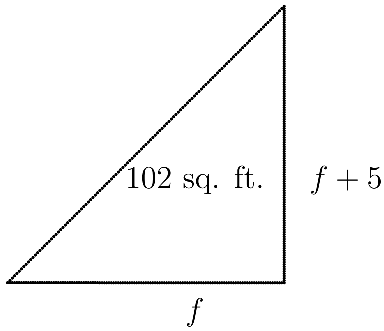
   **Answer:**




$95 = (\frac{1}{2})f(f+9)$
$(2)95 = (2)(\frac{1}{2})f(f+9)$
$190 = f^2 + 9f$
$\underline{\quad -190 \qquad -190}$
$0 = f^2 + 7f - 190$
$0 = (f-10)(f+19)$
$f_a = 10$
$f_b = -19$ extraneous root

3. The vertical edge of a triangular sail must be 5 feet longer than the foot of the sail. What is the length of the vertical edge if the area is 102 square feet?

**Answer:**




$$
\begin{array}{l}
102 = (\tfrac{1}{2})f(f+5) \\
(2)102 = (2)(\tfrac{1}{2})f(f+5) \\
204 = f^2 + 7f \\
\underline{\qquad -204 \qquad\quad -204} \\
0 = f^2 + 7f - 204 \\
0 = (f-12)(f+17) \\
f_a = 12 \\
f_b = -17 \text{ extraneous root}
\end{array}
$$

## 5.5 Example 3: Business Application

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *15. Applications of quadratic equations* in the MAT096 Review website, and watch the streaming video for *3) Business Application*. Check your work with the answers on the following page.

1. The cost and revenue function are given by $C(x) = 100 - 5x^2$ and $R(x) = 80x$. How many units must be sold to make a profit of \$60.

2. The cost and revenue function are given by $C(x) = 20 - 2x^2$ and $R(x) = 40x$. How many units must be sold to break even?

3. The cost and revenue function are given by $C(x) = 60 - 4x^2$ and $R(x) = 40x$. How many units must be sold to make a profit of \$1000.

## 5.6 Answers to Example 3: Business Application

1. The cost and revenue function are given by $C(x) = 100 - 5x^2$ and $R(x) = 80x$. How many units must be sold to make a profit of \$60.
   **Answer:**

$$
\begin{array}{l}
P = R - C \\
60 = 80x - (100 - 5x^2) \\
60 = 80x - 100 + 5x^2 \\
\underline{\quad -60 \qquad -60} \\
(\frac{1}{5})0 = (5x^2 + 80x - 160)(\frac{1}{5}) \\
0 = x^2 + 16x - 52 \\
\underline{\quad +52 \qquad +52} \\
52 = x^2 + 16x \\
\underline{\quad +64 \qquad +64} \\
116 = x^2 + 16x + 64 \\
\sqrt{116} = \sqrt{(x+8)^2} \\
\pm\sqrt{116} = x + 8 \\
\underline{\quad -8 \qquad -8} \\
-8 \pm \sqrt{116} = x \\
x_a = -8 + \sqrt{116} \\
x_b = -8 - \sqrt{116} \text{ extraneous root}
\end{array}
$$

2. The cost and revenue function are given by $C(x) = 20 - 2x^2$ and $R(x) = 40x$. How many units must be sold to break even?
   **Answer:**

$$
\begin{array}{l}
P = R - C \\
0 = 40x - (20 - 2x^2) \\
0 = 40x - 20 + 2x^2 \\
(\frac{1}{2})0 = (2x^2 + 40x - 20)(\frac{1}{2}) \\
0 = x^2 + 20x - 10 \\
\underline{\quad +10 \qquad +10} \\
10 = x^2 + 20x \\
\underline{\quad +100 \qquad +100} \\
110 = x^2 + 20x + 100
\end{array}
$$

$\sqrt{110} = \sqrt{(x+10)^2}$
$\pm\sqrt{110} = x + 10$
$\underline{\quad -10 \qquad -10}$
$-10 \pm \sqrt{116} = x$
$x_a = -10 + \sqrt{110}$
$x_b = -10 - \sqrt{110}$ extraneous root

3. The cost and revenue function are given by $C(x) = 60 - 4x^2$ and $R(x) = 40x$. How many units must be sold to make a profit of \$1000.
**Answer:**

$P = R - C$
$1000 = 40x - (60 - 4x^2)$
$1000 = 40x - 60 + 4x^2$
$\underline{\quad -1000 \qquad -1000}$
$(\frac{1}{4})0 = (4x^2 + 40x - 1060)(\frac{1}{4})$
$0 = x^2 + 10x - 265$
$\underline{\quad +265 \qquad +265}$
$265 = x^2 + 10x$
$\underline{\quad +25 \qquad +25}$
$290 = x^2 + 16x + 25$
$\sqrt{290} = \sqrt{(x+5)^2}$
$\pm\sqrt{290} = x + 5$
$\underline{\quad -5 \qquad -5}$
$-6 \pm \sqrt{290} = x$
$x_a = -6 + \sqrt{290}$
$x_b = -6 - \sqrt{290}$ extraneous root

## 5.7 Example 4: Physics Application 1

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *15. Applications of quadratic equations* in the MAT096 Review website, and watch the streaming video for *4) Physics Application 1.* Check your work with the answers on the following page.

1. $h(t) = -\frac{1}{2}t^2 - 2t + 1 = 0$ describes the height of a projectile after $t$ seconds. At what time will the height equation equal zero?

2. $h(t) = -\frac{1}{2}t^2 - 3t + 1 = 0$ describes the height of a projectile after $t$ seconds. At what time will the height equation equal zero?

3. $h(t) = -\frac{1}{2}t^2 + 2t - 2 = 0$ describes the height of a projectile after $t$ seconds. At what time will the height equation equal zero?

## 5.8 Answers to Example 4: Physics Application 1

1. $h(t) = -\frac{1}{2}t^2 - 2t + 1 = 0$ describes the height of a projectile after $t$ seconds. At what time will the height equation equal zero?
   **Answer:**

   $h(t) = -\frac{1}{2}t^2 - 2t + 1 = 0$
   $(2)0 = (2)(-\frac{1}{2}t^2 - 2t + 1)$
   $(-1)0 = (-1)(-t^2 - 4t + 2)$
   $0 = t^2 + 4t - 2$
   $\underline{\quad +2 \qquad +2}$
   $2 = t^2 + 4t$
   $\underline{\quad +4 \qquad +4}$
   $6 = t^2 + 4t + 4$
   $\sqrt{6} = \sqrt{(t+2)^2}$
   $\pm\sqrt{6} = t + 2$
   $\underline{\quad -2 \qquad -2}$
   $-2 \pm \sqrt{6} = t$
   $t_a = .44948974278318$
   $t_b = -2 - \sqrt{6}$ extraneous root

2. $h(t) = -\frac{1}{2}t^2 - 3t + 1 = 0$ describes the height of a projectile after $t$ seconds. At what time will the height equation equal zero?

   **Answer:**

   $h(t) = -\frac{1}{2}t^2 - 3t + 1 = 0$
   $(2)0 = (2)(-\frac{1}{2}t^2 - 3t + 1)$
   $(-1)0 = (-1)(-t^2 - 6t + 2)$
   $0 = t^2 + 6t - 2$
   $\underline{\quad +2 \qquad +2}$
   $2 = t^2 + 6t$
   $\underline{\quad +9 \qquad +9}$
   $11 = t^2 + 4t + 9$
   $\sqrt{11} = \sqrt{(t+3)^2}$

$\pm\sqrt{11} = t + 3$

$$\underline{\quad -3 \qquad -3}$$

$-3 \pm \sqrt{11} = t$

$t_a = .3166247903554$

$t_b = -3 - \sqrt{11}$ extraneous root

3. $h(t) = -\frac{1}{2}t^2 + 4t + 1 = 0$ describes the height of a projectile after $t$ seconds. At what time will the height equation equal zero?

**Answer:**

$h(t) = -\frac{1}{2}t^2 + 4t + 1 = 0$

$(2)0 = (2)(-\frac{1}{2}t^2 + 4t + 1)$

$(-1)0 = (-1)(-t^2 + 8t + 2)$

$0 = t^2 - 8t - 2$

$$\underline{\quad +2 \qquad +2}$$

$2 = t^2 - 8t$

$$\underline{\quad +16 \qquad +16}$$

$18 = t^2 - 8t + 16$

$\sqrt{18} = \sqrt{(t-4)^2}$

$\pm\sqrt{18} = t - 4$

$$\underline{\quad +4 \qquad +4}$$

$4 \pm \sqrt{18} = t$

$t_a = 4 + \sqrt{18}$

$t_b = 4 - \sqrt{18}$ extraneous root

## 5.9 Example 5: Physics Application 2

**Directions:** Go to http://faculty.lagcc.cuny.edu/gmccormack/ and click on *15. Applications of quadratic equations* in the MAT096 Review website, and watch the streaming video for *5) Physics Application 2*. Check your work with the answers on the following page.

1. With the current a swimmer goes three miles per hour faster downstream than upstream. If one trip is two miles, and the total time for a round trip is 4 hours, what is the speed of the swimmer in still water?

2. $h(t) = -\frac{1}{2}t^2 - 3t + 1 = 0$ describes the height of a projectile after $t$ seconds. At what time will the height equation equal zero?

3. $h(t) = -\frac{1}{2}t^2 + 2t - 2 = 0$ describes the height of a projectile after $t$ seconds. At what time will the height equation equal zero?

## 5.10 Answers to Example 5: Physics Application 2

1. With the current a swimmer goes three miles per hour faster downstream than upstream. If one trip is two miles, and the total time for a round trip is 4 hours, what is the speed of the swimmer in still water?
   **Answer:**

   *Let* $s$ = *the speed on the swimmer in still water* $4 = \frac{2}{s-3} + \frac{2}{s+3}$
   $(s+3)(s-3)4 = (s+3)(s-3)\frac{2}{s-3} + (s+3)(s-3)\frac{2}{s+3}$
   $(s+3)(s-3)4 = (s+3)2 + (s-3)2$
   $4s^2 + 36 = 2s + 6 + 2s - 6$
   $4s^2 - 36 = 4s$
   $\underline{\quad -4s \qquad -4s}$
   $\frac{1}{4}(4s^2 - 4s - 36) = (0)\frac{1}{4}$
   $s^2 - s + 9 = 0$
   $\underline{\quad +9 \qquad +9}$
   $s^2 - s = 9$
   $\underline{\quad \frac{1}{4} \qquad \frac{1}{4}}$
   $s^2 - s + \frac{1}{4} = 9 + \frac{1}{4}$
   $\sqrt{(s - \frac{1}{2})^2} = \sqrt{\frac{37}{4}}$
   $(s - \frac{1}{2} = \pm\sqrt{\frac{37}{4}}$
   $\underline{\quad \frac{1}{2} \qquad \frac{1}{2}}$
   $s = \frac{1 \pm \sqrt{37}}{2}$
   $s_a = \frac{1+\sqrt{37}}{2}$
   $s_b = \frac{1-\sqrt{37}}{2}$ extraneous root

2. Working together, Bill and Tom painted a fence in 8 hours. Last year, Tom painted the fence by himself. The year before, Bill painted it by himself, but took 12 hours less than Tom took. How long did Bill and Tom take, when each was painting alone?

   **Answer:**
   Total job takes 8 hours. Let Bill's time alone be $x$
   Let Tom's time alone be $x + 12$
   Then, the *part* of the job Bill can finish in 1 hour is $\frac{1}{x}$

and, the *part* of the job Tom can finish in 1 hour is $\frac{1}{x+12}$.
The part of the job that is completed in 1 hour is $\frac{1}{8}$.

$\frac{1}{x} + \frac{1}{x+12} = \frac{1}{8}$
$(8x(x+12))\frac{1}{x} + (8x(x+12))\frac{1}{x+12} = (8x(x+12))\frac{1}{8}$
$8(x+12) + 8x = x(x+12)$
$8x + 96 + 8x = x^2 + 12x$
$16x + 96 = x^2 + 12x$
$\underline{\quad -16x \qquad - 16x}$
$96 = x^2 - 4x$
$\underline{\quad -96 \qquad - 96}$
$0 = x^2 - 4x - 96$
$0 = (x-12)(x+8)$
$x_a = 12$
$x_b = -8$ extraneous root

3. The intensity $I$ of light, as measured in foot-candles, $x$ feet from its source is given by thee rational equation $I = \frac{320}{x^2}$. How for away is the source if the intensity of light 5 foot-candles?

**Answer:**
$I = \frac{320}{x^2}$
$5 = \frac{320}{x^2}$
$5x^2 = 320$
$\underline{\quad -320 \qquad - 320}$
$5x^2 - 320 = 0$
$5(x^2 - 64) = 0$
$(\frac{1}{5})5(x^2 - 64) = 0(\frac{1}{5})$
$(x+8)(x-8) = 0$

$x_a = 8$
$x_b = -8$ extraneous root